EREMS2: an Estimated Rational Expectation Model for Simulation and historical decomposition of the Spanish economy

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Abstract

In this paper we develop and estimate a new bayesian DSGE model for the Spanish economy that has been designed to evaluate different structural reforms. The small open economy model incorporates a banking sector, consumers and entrepreneurs that accumulate debt, a rich fiscal structure and monopolistic competition in products and labor markets, for a country in a currency union, with no independent monetary policy. The model can be used to evaluate ex-ante and ex-post policies and structural reforms and to decompose the evolution of macroeconomic aggregates according to different shocks.

Keywords: collateral constraints, banks, bank capital, fiscal policy, sticky interest rates.


1. Introduction

[TO BE COMPLETED]
2. Model description

The structure of the economy is as follows. We consider an open economy in which the home country (Spain) is relatively small to rest of the world, whose equilibrium is in the limit taken as exogenous (see, for example, Monacelli, 2003, or Galí and Monacelli, 2005). The home economy belongs to a monetary union with a central bank controlling the nominal interest rate. The home economy trades with the rest of the world (in our case the monetary union) consumption and investment goods as well as international nominal bonds. There are three types of households: patient, impatient, and hand-to-mouth. The patient (impatient) households consume, save (borrow), supply labor, and accumulate housing services. The hand-to-mouth households consume and have access neither to deposits nor to loans. Households delegate their labor decisions to labor unions who operate in monopolistically competitive markets. There are entrepreneurs whose main role is to purchase capital and rent it to intermediate good producers. In addition, entrepreneurs consume and borrow. Entrepreneurs buy capital from capital good producers who produce capital goods using final goods as inputs. Intermediate good producers hire labor from patient, impatient, and hand-to-mouth households, and rent capital from entrepreneurs in order to produce intermediate goods that are then sold to good retailers. Retailers buy intermediate goods and relabel them at no cost in order to sell them to consumers and capital producers in a monopolistically competitive market. The banking sector of the economy operates as follows. Banks form holding units each composed by three branches: a wholesale bank, a loan-retailing bank, and a deposit-retailing bank. Patient households deposit their savings on deposit-retailing banks. Impatient households and entrepreneurs take loans on loan-retailing banks. Deposit-retailing and loan-retailing banks operates in monopolistically competitive markets. The wholesale bank manages the capital position of the holding and intermediates funds between the deposit-retailing banks and the loan-retailing banks while counting on the monetary authority to fully-allotting their funding requirements at the policy rate. The source of monopolistic competition lies in the presence of differentiated types of labor, retail goods, deposits and loans, but for avoiding the complexities from introducing them explicitly in household preferences and good production technology, it is postulated the existence of intermediary "packers" who buy the different types of commodities and bundle or pack them and sell the homogeneous bundle to households and firms correspondingly.

Although implicitly there is at the union level a monetary authority that fixes the one-period nominal interest rate using a Taylor rule and supplyls full-allotment refinancing to wholesale banks, following Schmitt-Grohé and Uribe (2003), to ensure stationarity of equilibrium we assume that banks pay a risk-premium that increases with the country’s net foreign asset position. Thus, we close the model by assuming that the foreign borro-
wing interest rate is equal to an exogenous interest rate multiplied by a risk premium. Finally, there is a fiscal authority that consumes, invests, borrows or lends, sets lump-sum taxes, and taxes consumption, housing services, labor earnings, capital earnings, bond holdings, and deposits.

2.1 Patient Households
There is a continuum of patient households in the economy indexed by \( j \), with mass \( \gamma_p \), whose utility depends on consumption, \( c^p_{j,t} \); housing services, \( h^p_{j,t} \); and hours worked, \( \ell^p_{j,t} \), and has the following form:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - a_{cp}) \varepsilon^p_t \log(c^p_{j,t} - a_{cp}c^p_{j,t-1}) + a_{hp} \varepsilon^h_t \log(h^p_{j,t}) - \frac{a_{p/\ell^p_{j,t}}}{1 + \phi} \right],
\]

where \( c^p_t \) denotes average patient households consumption, i.e,

\[
c^p_t = \gamma_p^{-1} \left( \int_0^\gamma_p c^p_{j,t} dj \right);
\]

\( \varepsilon^p_t \) is a shock to consumption preferences with law motion

\[
\log \varepsilon^p_t = (1 - \rho_z) \log \varepsilon^p_{ss} + \rho_z \log \varepsilon^p_{t-1} + \sigma_z \varepsilon^p_t \quad \text{where } \varepsilon^p_t \sim \mathcal{N}(0,1);
\]

and \( \varepsilon^h_t \) is a shock to housing preferences with law of motion

\[
\log \varepsilon^h_t = (1 - \rho_h) \log \varepsilon^h_{ss} + \rho_h \log \varepsilon^h_{t-1} + \sigma_h \varepsilon^h_t \quad \text{where } \varepsilon^h_t \sim \mathcal{N}(0,1).
\]

The \( j \)th patient household is subject to the following budget constraint (expressed in terms of final goods):

\[
(1 + \tau^c_{j,t}) c^p_{j,t} + (1 + \tau^h_{j,t}) d^h_{j,t} \Delta h^p_{j,t} + d^p_{j,t} + \tau^d_{j,t} \Delta d^p_{j,t} =
\]

\[
(1 - \tau^w_{j,t}) w^p_{j,t} \ell^p_{j,t} + \left[ \frac{1+(1-\tau^w_{j,t}) t_{j,t-1}}{\tau^w_{j,t}} \right] d^p_{j,t-1} + \frac{\tau^b_{j,t}}{\tau^b_{j,t}} + (1 - \omega_b) \frac{\tau^w_{j,t}}{\tau^w_{j,t}} - \frac{\tau^w_{j,t}}{\tau^w_{j,t} + \tau^h_{j,t} + \tau^d_{j,t}}.
\]

where \( \pi_t = \frac{P_t}{P_{t-1}} \) is gross inflation with \( P_t \) denoting the price level of final goods and the variables \( \tau^c_{j,t}, \tau^h_{j,t}, \tau^d_{j,t} \) denote taxes on consumption, accumulation of housing services, interest income from deposits, and deposit transactions, respectively. The flow of expenses, expressed in terms of final goods, are consumption (plus consumption taxes),
(1 + τ_f^c) c_{j,t}^p; accumulation of housing services (plus housing services taxes), (1 + τ^h_f) q_{j,t}^h Δh_{j,t}^p; current deposits, d_{j,t}^p, and taxes on the variation in the stock of deposits, τ_d^f Δd_{j,t}^p. The sources of income are labor income (minus wage income taxes), (1 - τ_t^p) w_{j,t}^p e_{j,t}^p; deposits gross return from the previous period (minus interest income taxes), \left[\frac{1 + (1 - τ_d^p) r_{t-1}^d}{π_t}\right] d_{j,t-1}^p; dividends from the retail firms, \frac{r_{j,t}^p}{π_p}; dividends from the banking sector, (1 - \omega^p) \frac{R_{j,t}^p}{π_p}, net of lump-sum fees paid to the unions, \frac{τ_{up}^p}{π_p}; and lump-sum taxes paid to the government, \frac{τ_{up}^g}{π_p + γ_t + γ_e}.

Households have access to a Arrow-Debreu securities. We do not write the whole set of possible Arrow-Debreu securities in the budget constraint to save on notation. Since their net supply is zero, they are not traded in equilibrium. However, households could trade and price any of these securities.

The representative patient household chooses c_{j,t}^p, d_{j,t}^p, h_{j,t}^p (decision on \omega_{j,t}^p and \lambda_{j,t}^p, is delegated on a "labor union" whose decision is described below) for t = 0, 1, 2, ..., +∞ in order to maximize utility subject to budget constraint. The lagrangian of this maximization problem is

\[ E_0 \sum_{t=0}^{+∞} B^t \left[ (1 - a_{cp}) e_{j,t}^p \log(c_{j,t}^p - a_{cp} c_{j,t-1}^p) + a_{hp} e_{j,t}^p \log(h_{j,t}^p) - \frac{a_{lp} e_{j,t}^{1+p}}{1 + φ} - λ_{j,t}^p \left\{ (1 + τ_f^c) c_{j,t}^p + (1 + τ^h_f) q_{j,t}^h Δh_{j,t}^p + (1 + τ_d^f) Δd_{j,t}^p - (1 - τ_t^p) \omega_{j,t}^p c_{j,t}^p - \left[ \frac{1 + (1 - τ_d^p) r_{t-1}^d}{π_t}\right] d_{j,t-1}^p - \frac{r_{j,t}^p}{π_p} - (1 - \omega^p) \frac{R_{j,t}^p}{π_p} + \frac{τ_{up}^p}{π_p} + \frac{τ_{up}^g}{γ_p + γ_t + γ_e} \right\} \right]. \]

and the corresponding first order conditions are

\[ λ_{j,t}^p (1 + τ_c) - \frac{(1 - a_{cp}) e_{j,t}^p}{c_{j,t}^p - a_{cp} c_{j,t-1}^p} = 0 \]
\[ \frac{a_{hp} e_{j,t}^h}{h_{j,t}^p} - λ_{j,t}^p (1 + τ_h^f) q_{j,t}^h + β_p E_t \left\{ λ_{j,t+1}^p (1 + τ_h^f) q_{j,t+1}^h \right\} = 0 \]
\[ λ_{j,t}^p (1 + τ_{fd}) - β_p E_t \left\{ λ_{j,t+1}^p \left[ \frac{1 + (1 - τ_d^p) r_{t-1}^d}{π_t+1} + τ_{fd} \right] \right\} = 0 \]
2.2 Labor and wage decisions

Given the similarity of the problem of choosing wages and labor supply for the three types of households, we present a general derivation of the problem using the superindex $s$ to denote patient households, $s = p$; impatient households, $s = i$; and hand-to-mouth households, $s = m$; so in the following description the $j$th $s$-type household is denoted by the index pair $(j, s)$ accordingly.

Each household delegates its labor decision to $\ell$abor unions (one per each possible value of $(j, s)$). Unions sell labor, in a monopolistically competitive market, to three $\ell$abor packers (one for each category of household). Packers finally sell it to intermediate good producers after bundling it with the hours supplied by the rest of households of the same type, using the following production function:

$$
\ell_s^j = \left( \int_0^{\gamma_s} \left( \ell_{j, s}^{s-1} \right)^{\epsilon_s^j} dj \right)^{\epsilon_s^j},
$$

where $\ell_s^j$ is the per-household aggregate demand for labor from households of type $s$ and $\epsilon_s^j$ is the elasticity of substitution among different types of labor which is stochastic and follows the law of motion

$$
\log(\epsilon_s^j) = (1 - \rho_s) \log(\epsilon_s^{s-1}) + \rho_s \log(\epsilon_{s-1}^j) + \sigma_s \epsilon_s^j \epsilon_s^{s-1} + \epsilon_s^j \epsilon_s^{s-1} e_t \epsilon_s^j
$$

where $e_t \sim N(0, 1)$.

Then, the representative labor packer chooses the demand for $\ell_{j, s}^s$ for all $j \in \gamma_s$ in order to maximize

$$
w_s^j \ell_s^j - \int_0^{\gamma_s} w_{j, s}^s \ell_{j, s}^s dj.
$$

subject to its production function and taking as given all differentiated labor wages, $w_{j, s}^s$ for all $j \in \gamma_s$, and their aggregate, $w_s^j$ (defined below).

The corresponding first order condition is:

$$
w_s^j \frac{\epsilon_s^j}{\epsilon_s^j - 1} \left( \frac{\epsilon_s^{s-1}}{\epsilon_s^j} \right)^{\epsilon_s^{s-1} - 1} \frac{\epsilon_s^j - 1}{\epsilon_s^j} \left( \epsilon_{j, s}^s \right)^{\epsilon_s^j - 1} - w_{j, s}^s = 0 \quad \text{for all } j \in \gamma_s
$$

and dividing the first order conditions for two members $s_i$ and $s_j$ of the $s$-type household group, we obtain
\[ \frac{w^s_{s_i,t}}{w^s_{s_j,t}} = \left( \frac{\ell^s_{s_i,t}}{\ell^s_{s_j,t}} \right)^{-\frac{1}{\epsilon^i}} \]

or

\[ w^s_{s_j,t} = \left( \frac{\ell^s_{s_i,t}}{\ell^s_{s_j,t}} \right)^{\frac{1}{\epsilon^i}} w^s_{s_i,t} \]

hence,

\[ w^s_{s_j,t} \ell^s_{s_j,t} = w^s_{s_i,t} \left( \ell^s_{s_i,t} \right)^{\frac{1}{\epsilon^i}} \left( \ell^s_{s_j,t} \right)^{\frac{\epsilon^j-1}{\epsilon^i}} \]

and integrating out,

\[ \int_0^{\gamma^s} w^s_{s_j,t} \ell^s_{s_j,t} ds_j = w^s_{s_i,t} \left( \ell^s_{s_i,t} \right)^{\frac{1}{\epsilon^i}} \int_0^{\gamma^s} \left( \ell^s_{s_j,t} \right)^{\frac{\epsilon^j-1}{\epsilon^i}} ds_j = w^s_{s_i,t} \left( \ell^s_{s_i,t} \right)^{\frac{1}{\epsilon^i}} \left( \ell^s_{s_j,t} \right)^{\frac{\epsilon^j-1}{\epsilon^i}}, \]

but, by the zero profits condition implied by perfect competition, i.e, \( w^s_{i,t} = \int_0^{\gamma^s} w^s_{j,t} \ell^s_{j,t} ds_j \), we get

\[ w^s_{i,t} \ell^s_{i,t} = w^s_{s_i,t} \left( \ell^s_{s_i,t} \right)^{\frac{1}{\epsilon^i}} \left( \ell^s_{i,t} \right)^{\frac{\epsilon^j-1}{\epsilon^i}} \Rightarrow w^s_{i,t} = w^s_{s_i,t} \left( \ell^s_{s_i,t} \right)^{\frac{1}{\epsilon^i}} \left( \ell^s_{i,t} \right)^{-\frac{1}{\epsilon^i}} \]

and, as a result, given that the index \( s_j \) is arbitrary the input demand functions associated with this problem are

\[ \ell^s_{j,t} = \left( \frac{w^s_{j,t}}{w^s_{i,t}} \right)^{-\frac{1}{\epsilon^j}} \ell^s_{i,t} \quad \text{for all } j \in \gamma^s. \]

To find the aggregate wage for each type of labor, we use again the zero profit condition \( w^s_{i,t} \ell^s_{i,t} = \int_0^{\gamma^s} w^s_{j,t} \ell^s_{j,t} ds_j \) and plug-in the input demand functions:

\[ w^s_{i,t} \ell^s_{i,t} = \int_0^1 w^s_{j,t} \left( \frac{w^s_{j,t}}{w^s_{i,t}} \right)^{-\epsilon^j} \ell^s_{i,t} ds_j \Rightarrow w^s_{i,t}^{1-\epsilon^j} = \int_0^1 w^s_{j,t}^{1-\epsilon^j} ds_j \]

which yields

\[ w^s_{i,t} = \left( \int_0^{\gamma^s} w^s_{j,t}^{1-\epsilon^j} ds_j \right)^{\frac{1}{1-\epsilon^j}}. \]
Now, as explained before, unions intermediate labor services between households and packers, specifically, the representative union, denoted by the pair \((s,j)\), sets the nominal wage for its type of labor by maximizing the following objective function, which represents the utility perceived by the household supplying the labor from the resulting wage income net of a quadratic cost for adjusting the nominal wage,

\[
E_0 \sum_{t=0}^{+\infty} \beta_s^t \left\{ U_{c_{j,t}}^{s} \theta_t^{wc} \left[ w_{j,t}^s \ell_{j,t}^s - \frac{\eta_w}{2} \left( \pi_{j,t}^{ws} \theta_t^{w} - \pi_{t-1}^{w} \pi_{t-1}^{1-\theta_{t-1}} \right)^2 W_{j,t}^s \right] - \frac{a_{fs} \ell_{j,t}^{1+\phi}}{1 + \phi} \right\}
\]

subject to

\[
\ell_{j,t}^s = \left( \frac{w_{j,t}^s}{w_t^s} \right)^{-\ell_t^s}
\]

\[
w_{j,t}^s = \frac{W_{j,t}^s}{P_t}
\]

where \(\pi_{j,t}^{ws} \equiv \left( \frac{w_{j,t}^s}{w_{j,t-1}^s} \right) \pi_{t,s} \), \(\theta_t^{wc} \equiv \left( \frac{1-\tau_t^p}{1+\tau_t^p} \right) \), \(\theta_t^w \equiv \left( \frac{1-\tau_t^m}{1+\tau_t} \right) \), \(\theta_t^c \equiv \left( \frac{1+\tau_t}{1-\tau_t} \right) \).

\(U_{c_{j,t}}^{s}\) represents the instantaneous marginal utility taken as given by unions. So, denoting \(U_{j,t}^{s}\) as the instantaneous total utility function, i.e,

\[
U_{j,t}^{s} \equiv \begin{cases} 
(1 - a_{cs}) e_{j,t}^s \log(c_{j,t}^s - a_{cs} c_{t-1}^s) + a_{hs} e_{j,t}^s \log(h_{j,t}^s) - \frac{a_{fs} \ell_{j,t}^{1+\phi}}{1 + \phi} & \text{for } s = p, i \\
(1 - a_{cs}) e_{j,t}^s \log(c_{j,t}^s - a_{cs} c_{t-1}^s) - \frac{a_{fs} \ell_{j,t}^{1+\phi}}{1 + \phi} & \text{for } s = m
\end{cases}
\]

then,

\[
U_{c_{j,t}}^{s} \equiv \frac{\partial U_{j,t}^{s}}{\partial c_t} = \frac{(1 - a_{cs}) e_{j,t}^s}{c_{j,t}^s - a_{cs} c_{t-1}^s}.
\]

Later will be shown that in equilibrium \(U_{j,t}^{s} = (1 + \tau_t^p) \lambda_{j,t}^{s}\) for \(s = p, i\). After substituting the constraint into the objective function, the FOC of the Union with respect
to type $p$ wage yields

$$\left[(1 - c_i^p)\ell_i^p - \eta_w \left\{ \pi_{j,t}^{wp} + (1 - \omega) \pi_{j,t-1} \right\} \right] + \frac{a_{p}^{e} \ell_i^{1+p}}{\theta_p(1 - \tau_w) w_i^p} +$$

$$\beta_p E_t \left\{ \frac{\lambda_{t+1}^p}{\lambda_t^p} \left[ \eta_w \left( \pi_{j,t+1}^{wp} - \pi_{j,t-1}^{wp} \right) \right] \right\} = 0.$$  

Since we assume complete markets, we can consider a symmetric equilibrium where $c_i^p = c_{t,t}^p, h_i^p = h_{t,t}^p, d_t = d_{t,t}^p, \lambda_t^p = \lambda_{t,t}^p,$ and $U_t = U_{i,t}$. In addition, following Rotemberg (1982) we construct a symmetric equilibrium where the wage set by each household is optimal, given the wages set by other households as well as the expectation of wages that other households will set in the future. As a consequence, $w_i^t = w_{j,t}^t$. Thus, the first order conditions associated with the patient households’ problem becomes:

$$\lambda_t^p (1 + \tau_c) - \left\{ \frac{(1 - a_{p}^{e}) \ell_i^p}{c_i^p - a_{p}^{e} \ell_i^{p-1}} \right\} = 0$$

$$\frac{a_{p}^{e} \ell_i^{p}}{h_i^p} - \lambda_t^p (1 + \tau_h) q_t^h + \beta_p E_t \left\{ \lambda_{t+1}^p \left[ 1 + (1 - \tau_d) \right] \right\} = 0$$

$$\lambda_t^p (1 + \tau_d) - \beta_p E_t \left\{ \lambda_{t+1}^p \left[ \frac{1 + (1 - \tau_d) \right] \right\} + \left[ (1 - c_i^p) \ell_i^p - \eta_w \left( \pi_{t}^{wp} - \pi_{t-1}^{wp} \right) \right] + \frac{a_{p}^{e} \ell_i^{1+p}}{\lambda_t^p (1 - \tau_w) w_i^p} +$$

$$\beta_p E_t \left\{ \frac{\lambda_{t+1}^p}{\lambda_t^p} \left[ \eta_w \left( \pi_{t+1}^{wp} - \pi_{t}^{wp} \right) \right] \right\} = 0,$$

where $\pi_{t}^{wp} = \frac{w_i^p}{\omega_{i-1}} \pi_t$. Using the zero profits condition of the labor packer, $w_i^p = \frac{\gamma_{p} w_{j,t} \ell_i^p d_t}{\gamma_{p} w_{j,t} \ell_i^p d_t},$ the budget constraint of the patient households can be written as:

$$(1 + \tau_i^p) c_i^p + (1 + \tau_i^p) q_i^p \Delta h_i^p + (1 + \tau_i^{fd}) d_i^p =$$

$$(1 - \tau_i^p) w_i^p \ell_i^p + \left[ \frac{1 + (1 \tau_i^{fd})}{\pi_t} \right] d_i^{p-1} + \frac{\gamma_{p} \ell_i^p}{\gamma_{p}} + (1 - \omega_\ell) \frac{h_i^p}{\gamma_{p}} - \frac{T_{p}^{wp}}{\gamma_{p}} - \frac{T_{p}^f}{\gamma_{p} + \gamma_{p} + \gamma_{e}}.$$  

Finally, the cost of participating in the labor union are equal to the quadratic cost of
changing the wage

\[ T_{i}^{up} = \gamma_p \frac{\eta_w}{2} \left( \pi_t^{wp} \theta_t^{wp} - \pi_{i-1}^{pw} \pi_{i-1}^{1-\iota w} \theta_{i-1}^{wp} \right)^2 w_t \]

### 2.3 Impatient Households

There is a continuum of impatient households in the economy indexed by \( j \), with mass \( \gamma_j \), whose utility function depends on consumption \( c_{i,j,t} \), housing services \( h_{i,j,t} \) and hours worked \( \ell_{i,j,t} \), and has the following form:

\[
E_0 \sum_{t=0}^{\infty} \beta_t \left[ (1 - a_{c})\epsilon_t^c \log(c_{i,j,t}) - a_{h}\epsilon_t^h \log(h_{i,j,t}) - \frac{a_{\ell}\epsilon_t^\ell + \phi}{1 + \phi} \right]
\]

where \( c_t^i \) denote average patient households consumption, i.e,

\[ c_t^i = \gamma_t^{-1} \left( \int_0^{T_t} c_{i,j,t} dj \right) \]

and \( \epsilon_t^c \) and \( \epsilon_t^h \) are defined as in the patient household problem above. The \( j \)th impatient household budget constraint, expressed in terms of final goods, is given by:

\[
(1 + \tau_c^c)c_{i,j,t} + (1 + \tau_h^h)q_{i,j,t} \Delta h_{i,j,t} + \left( \frac{1+\tau_f^h}{\pi_t} \right) b_{i,j,t-1} + \tau_f^h \Delta b_{i,j,t} =
\]

\[
(1 - \tau_w^w)w_{i,j,t} \ell_{i,j,t} + b_{i,j,t} - \frac{T_{ui}}{\gamma_i} - \frac{T_{ug}^p}{\gamma_p + \gamma_i + \gamma_e}.
\]

Impatient households do not hold deposits, and own neither the banks nor the retail firms. The flow of expenses of impatient households is given by consumption (plus consumption taxes) \((1 + \tau_c^c)c_{i,j,t}\), housing services (plus housing services taxes) \((1 + \tau_h^h)q_{i,j,t} \Delta h_{i,j,t}\), interest plus principal of loans taken during the previous period \(\left( \frac{1+\tau_f^h}{\pi_t} \right) b_{i,j,t-1}\), and taxes ad-valorem on new lending transactions. The sources of income are labor income (minus wage taxes) \((1 - \tau_w^w)w_{i,j,t} \ell_{i,j,t}\) and loans \(b_{i,j,t}\) net of lump-sum fees paid to the unions \(\frac{T_{ui}}{\gamma_i}\), and lump-sum taxes paid to the government \(\frac{T_{ug}^p}{\gamma_p + \gamma_i + \gamma_e}\).

In the last expression, under the same grounds that in the patient households case, we omit the whole set of Arrow-Dereu securities accesible to impatient households.

In addition, impatient households face a borrowing constraint: always in terms of final goods, they cannot borrow more than a certain proportion of the expected value of
their housing stock at period \( t \), i.e,

\[
(1 + r^t_i)b^i_{j,t} \leq m^i_t E_t \left\{ q^h_{t+1} h^i_{j,t+1} \pi_{t+1} \right\},
\]

where \( m^i_t \) is the stochastic loan-to-value ratio for mortgages with law of motion:

\[
\log m^i_t = (1 - \rho_{m^i}) \log m_{ss}^i + \rho_{m^i} \log m^i_{t-1} + \sigma_{m^i} \epsilon^m_{t}\] where \( \epsilon^m_t \sim \mathcal{N}(0,1) \).

We assume that the shocks in the model are small enough so that we can solve the model imposing that the borrowing constraint always binds as in Iacoviello (2005).

Impatient households choose \( c^i_t \), \( h^i_{j,t} \), and \( b^i_{j,t} \) (remember that the decision on labor and wages are delegated on the corresponding labor union) for \( t = 0, 1, 2, ..., +\infty \) in order to maximize its utility function subject to budget and credit constraints. The corresponding Lagrangian of this optimization problem is

\[
E_0 \sum_{t=0}^{+\infty} \beta^t_j \left[ \left( 1 - a_{ci} \right) c^i_{j,t} \log(c^i_{j,t} - a_{ci} c^i_{j,t-1}) + a_{hi} h^i_{j,t} \log(h^i_{j,t} - a_{hi} h^i_{j,t-1}) - \frac{a_{li} \epsilon^l_{j,t}}{1 + \phi} \right] + \lambda^i_{j,t} \left\{ (1 + \tau^c_t) c^i_{j,t} + (1 + \tau^h_t) h^i_{j,t} + \left( 1 + r^b_{t-1} \pi_{t-1} - \tau^b_t \right) b^i_{j,t-1} - \left( 1 + \tau^w_t \right) w^i_{j,t} - \left( 1 - \tau^b_t \right) b^i_{j,t} + \frac{T^u_{t}}{\gamma_t} + \frac{T^\pi_{t}}{\gamma_t + \gamma_c + \gamma_e} \right\} - \xi^i_{j,t} \left\{ (1 + r^b_t) b^i_{j,t} - m^i_t E_t \left\{ q^h_{t+1} h^i_{j,t+1} \pi_{t+1} \right\} \right\},
\]

and the associated first order conditions are

\[
\lambda^i_{j,t} (1 + \tau^c_t - \frac{(1 - a_{ci}) \epsilon^c_t}{c^i_{j,t} - a_{ci} c^i_{j,t-1}} = 0
\]

\[
\frac{a^{hi} c^h_{j,t}}{h^i_{j,t}} - \lambda^i_{j,t} (1 + \tau^h_t) q^h_{j,t} + \xi^i_{j,t} m^i_t E_t \left\{ q^h_{t+1} \pi_{t+1} \right\} + \beta^i_t E_t \left\{ \lambda^i_{j,t+1} (1 + \tau^h_t) q^h_{t+1+1} \right\} = 0
\]

\[
\lambda^i_{j,t} (1 - \tau^b_t) - \beta^i_t E_t \left\{ \lambda^i_{j,t+1} \left( \frac{1 + r^b_{t+1}}{\pi_{t+1}} - \tau^b_t \right) \right\} - \xi^i_{j,t} (1 + r^b_t) = 0.
\]

To determine the optimal values of \( w^i_{j,t} \) and \( \epsilon^i_{j,t} \), we should add the first order conditions of the union’ optimization problem, derived before, i.e,
can be expressed as

therefore, the first order conditions associated to the patient household's control variables

households can be written as

responding labor packers,

where

By the reasons stated in the description of the problem of patient households, we can focus on a symmetric equilibrium where $c_i^j = c_{i,t}^j$, $h_i^j = h_{i,t}^j$, $b_i^j = b_{i,t}^j$, $w_i^j = w_{i,t}^j$, therefore, the first order conditions associated to the patient household’s control variables can be expressed as

$$
\lambda_i^j(1 + \tau_c) - \frac{(1 - a^c_i)\varepsilon_i^j}{c_i^j - a^c_i c_{i-1}^j} = 0
$$

$$\frac{a^b_i c_i^j}{h_i^j} - \lambda_i^j(1 + \tau_h)q_i^h + \xi_i^j m_i^j E_t \left\{ q_i^{h_t+1} \pi_{t+1} \right\} + \beta_i E_t \left\{ \lambda_i^j(1 + \tau_h)q_i^{h_t+1} \right\} = 0
$$

$$\lambda_i^j(1 - \tau_f) - \beta_i E_t \left\{ \lambda_i^{j+1} \left( \frac{1 + r_i^{h_t}}{\pi_{t+1}} - \tau_f \right) \right\} - \xi_i^j(1 + r_i^{h_t}) = 0
$$

$$\left[ (1 - \varepsilon_i^j)\ell_i^j - \eta_w \left( \pi_i^{wi_t} - \pi_i^{w_{t-1}} \pi^{1-w_t} \right) \pi_i^{wi_t} \right] + \frac{a^\ell_i^j \ell_i^{1+\phi}}{\lambda_i^j(1 - \tau_w)w_i^j} + +

\beta_i E_t \left\{ \frac{\lambda_i^{j+1}}{\lambda_i^j} \left[ \eta_w \left( \pi_i^{wi_t} - \pi_i^{w_{t+1}} \pi^{1-w_t} \right) \pi_i^{wi_t+1} \pi_i^{wi_t+1} \pi^{1-w_t} \right] \right\} = 0,
$$

where $\pi_i^{wi_t} = \frac{w_i^j}{\pi_i^{wi_{t-1}}}.\pi_t.\

Also, assuming a symmetric equilibrium and the zero profits condition of the corresponding labor packers, $w_i^j \ell_i^j = \int_0^{\tau_i} w_{i,t} \ell_i^{j,t} dj$, the budget constraint of the impatient households can be written as

$$
(1 + \tau_f^i) c_i^j + (1 + \tau_i^h) q_i^h \Delta h_i^j + \left( \frac{1 + r_i^{h_t}}{\pi_t} - \tau_f^i \right) b_i^{j+1} = 0
$$

$$
(1 - \tau_i^w) w_i^j \ell_i^j + (1 - \tau_f^i) b_i^j - \frac{\pi_i^{wi_t}}{\pi_t} - \frac{\pi_i^{wi_t+1}}{\pi_t+1}.\pi_e
$$
and the binding borrowing constraint can be written as

\[(1 + r_{i}^{bi})b_{i} = m_{i}E_{i}\left\{ q_{i+1}^{bi}h_{i}^{i}i_{t+1}\right\} ;\]

Finally, the cost of participating in the labor union are equal to the quadratic cost of changing the wage

\[T_{j}^{\mu i} = \frac{\gamma_{j}^w}{2} \left( \pi_{j}^{\mu i}w_{i} - \pi_{j-1}^{\mu i}w_{i-1}\right)^2 w_{i}.\]

### 2.4 Hand-to-mouth Households

There is a continuum of hand-to-mouth households in the economy indexed by \(j\), with mass \(\gamma_{m}\), whose utility function depends on consumption \(c_{j,i}^{m}\), and hours worked \(\ell_{j,i}\), and has the following form:

\[E_{0}^{\infty} \sum_{t=0}^{\infty} \beta_{m}^{t} \left( 1 - a_{cm}c_{j,i}^{m}\log(c_{j,i}^{m} - a_{cm}c_{j-1,i}^{m}) - \frac{a_{cm}\ell_{j,i}^{m+1} + \phi}{1 + \phi} \right).\]

where \(c_{j}^{m}\) denote average hand-to-mouth households consumption, i.e,

\[e_{j,i}^{m} = \gamma_{m}^{-1} \left( \int_{0}^{\gamma_{m}} c_{j,i}^{m}d\gamma \right) ;\]

The \(j\)th hand-to-mouth household budget constraint is given by:

\[ (1 + \tau_{i}^{c})c_{j,i}^{m} = (1 - \tau_{i}^{w})w_{i}^{m}\ell_{i}^{m} - \frac{T_{j}^{\mu m}}{\gamma_{m}}. \]

Hand-to-mouth households do not hold deposits, do not get utility from housing services, and own neither the banks nor the retail firms. The only expense of hand-to-mouth households is consumption (plus consumption taxes) \((1 + \tau_{i}^{c})c_{j,i}^{m}\). The sources of income are labor income, \((1 - \tau_{i}^{w})w_{i}^{m}\ell_{i}^{m}\), net of lump-sum fees paid to the unions, \(\frac{T_{j}^{\mu m}}{\gamma_{m}}\).

Then, the Lagrangian of hand-to-mouth optimization problem is:

\[E_{0}^{\infty} \sum_{t=0}^{\infty} \beta_{m}^{t} \left( 1 - a_{cm}c_{j,i}^{m}\log(c_{j,i}^{m} - a_{cm}c_{j-1,i}^{m}) - \frac{a_{cm}\ell_{j,i}^{m+1} + \phi}{1 + \phi} \right) \]

\[\lambda_{j,i}^{m} \left\{ (1 + \tau_{i}^{c})c_{j,i}^{m} - (1 - \tau_{i}^{w})w_{i}^{m}\ell_{i}^{m} + \frac{T_{j}^{\mu m}}{\gamma_{m}} \right\} \]
where the households maximize over $c_{j,t}^m$ (remember that the decision over wages and labor is delegated on the corresponding labour union) for $t = 0, 1, 2, ..., +\infty$, resulting the following first order condition:

$$(1 + \tau_c) c_{j,t}^m = (1 - \tau_w) w_{j,t}^m \ell_{j,t}^m - \frac{T_t^{um}}{\gamma_m}.$$ 

The union’s first order conditions wich determine $w_{j,t}^m$ and $\ell_{j,t}^m$ follow the same logic as in the case of patient and impatient households with the obvious modifications, that is,

$$
\left(\frac{1 - \tau_w}{1 + \tau_c}\right) \left[(1 - \varepsilon_t^f) \ell_t^m - \eta_w \left(\pi_{t+1}^{wm} - \pi_{t-1}^{wm} \pi_t^{1-\varepsilon} \right) \right] + \frac{a_{cm} \ell_{j,t}^m \pi_t^{1+\alpha}}{U_{c,j,t}^m} \pi_t^{wm} + \beta_m \left(\frac{U_{c,j,t+1}^m}{U_{c,j,t}} \varepsilon_t^m \right) = 0
$$

where

$$U_{c,j,t}^m = \frac{(1 - a_{cm}) \varepsilon_t^m}{\ell_{j,t}^m - a_{cm} \ell_{j-1}^m}.$$ 

Similar to the case of patient and impatient households, we consider a symmetric equilibrium where $c_t^m = c_{j,t}^m$, $w_t^m = w_{j,t}^m$, and $\ell_t^m = \ell_{j,t}^m$. Therefore, the first order conditions associated to the hand-to-mouth household’s problem become:

$$(1 + \tau_c) c_t^m = (1 - \tau_w) w_t^m \ell_t^m - \frac{T_t^{um}}{\gamma_m}$$

$$
\left(\frac{1 - \tau_w}{1 + \tau_c}\right) \left[(1 - \varepsilon_t^f) \ell_t^m - \eta_w \left(\pi_{t+1}^{wm} - \pi_{t-1}^{wm} \pi_t^{1-\varepsilon} \right) \right] + \frac{a_{cm} \ell_t^m \pi_t^{1+\alpha}}{U_{c,t}^m} \pi_t^{wm} + \beta_m \left(\frac{U_{c,t+1}^m}{U_{c,t}} \varepsilon_t^m \right) = 0
$$

where $\pi_t^{wm} = \frac{w_t^m}{\ell_{t-1}^m}$ and

$$U_{c,t}^m = \frac{(1 - a_{cm}) \varepsilon_t^m}{\ell_t^m - a_{cm} \ell_{t-1}^m},$$

and the budget constraint, again by the packer’s zero profits condition, $w_t^m \ell_t^m = \int_0^{T_t} w_{j,t}^m \ell_{j,t}^m dj$, 

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becomes

\[(1 + \tau_c) c_t^m = (1 - \tau_w) w_t^m \ell_t^m - \frac{T_t^{um}}{\gamma_m}.\]

Finally, the cost of participating in the labor union are equal to the quadratic cost of changing the wage

\[T_t^{um} = \gamma_m \eta \frac{1}{2} \left( \frac{\eta_{t^m} \theta_{t^m}}{\eta_{t^m} \theta_{t^m}} - \pi_{t^m}^{1-\eta} \theta_{t^m}^{1-\eta} \right)^2 w_t^m.\]

### 2.5 Entrepreneurs

There is a continuum of entrepreneurs in the economy indexed by \(j\), with mass \(\gamma_e\), that choose consumption \(c_{j,t}^e\), capital \(k_{j,t}^e\), and loans \(b_{j,t}^e\) for \(t = 0, 1, 2, \ldots, +\infty\) to maximize the following lifetime utility function

\[
E_0 \sum_{t=0}^{+\infty} \beta^t (1 - a_e) \log (c_{j,t}^e - a_e c_{j-1}^e).
\]

where \(c_t^e\) denotes average entrepreneurs consumption, i.e,

\[c_t^e = \gamma_e^{-1} \left( \int_0^t c_{j,t}^e d\ell \right);\]

subject to following budget constraint:

\[\begin{aligned}
(1 + \tau_t^e) c_{j,t}^e &+ \left( \frac{1 + r_{t}^e}{\pi_t} \right) b_{j,t-1}^e + \tau_t^{fb} b_{j,t}^e + q_t k_{j,t}^e = \\
(1 - \tau_t^e) r_t^e k_{j,t}^e &+ b_{j,t}^e + q_t^e (1 - \delta) k_{j,t-1}^e + \frac{\mathbf{I}}{\tau_c} + \frac{\mathbf{R}}{\tau_r} - \frac{T_t^{fb}}{\tau_p},
\end{aligned}\]

So, entrepreneurs do not hold deposits, they own the intermediate good producers firms and the capital good producers firms, but own neither the banks nor the retail firms. The flow of expenses of entrepreneurs is given by consumption (plus consumption taxes) \((1 + \tau_t^e) c_{j,t}^e\), capital purchases \(q_t k_{j,t}^e\), interest plus principal of loans taken during the previous period \(\left( \frac{1 + r_{t-1}^e}{\pi_{t-1}} \right) b_{j,t-1}^e\), and taxes on new lending transactions, \(\tau_t^{fb} b_{j,t}^e\). The sources of income are rental capital (minus capital taxes), \((1 - \tau_t^e) r_t^e k_{j,t}^e\), loans, \(b_{j,t}^e\), capital from the previous period \(q_t^e (1 - \delta) k_{j,t-1}^e\), dividends from intermediate good producers
where $\frac{\ell}{\ell'}$ are dividends from capital good producers, $\frac{d}{d'}$, and net of lump-sum taxes paid to the government, $\frac{T_F}{\gamma_p+\gamma_i+\gamma_p}$.

In addition, entrepreneurs face a borrowing constraint: they cannot borrow more than a certain proportion of the expected value of their capital stock at period $t$, that is

$$(1 + r_l^{be})b_{j,t} \leq m_t^e E_t \left\{ q_{t+1}^k \pi_{t+1} (1 - \delta) k_{j,t}^e \right\},$$

where $m_t^e$ is the stochastic loan-to-value ratio for capital with law of motion:

$$\log m_t^e = (1 - \rho_{me}) \log m_{ss}^e + \rho_{me} \log m_{t-1}^e + \sigma_{me} e_t^m \quad \text{where } e_t^m \sim \mathcal{N}(0, 1).$$

As in the case of impatient households, we assume that the shocks in the model are small enough so that we can solve the model imposing that the borrowing constraint always binds. Then the Lagrangian is:

$$E_0 \sum_{t=0}^{\infty} \beta^t_c \left[ (1 - \alpha_c) \log(c_{j,t}^e - a_c c_{t-1}^e) - \lambda_{j,t}^e \left\{ (1 + \tau_{j,t}^e) c_{j,t}^e + \left( \frac{1 + r_l^{be}}{\pi_t} - \tau_t^{fb} \right) b_{j,t-1}^e + \xi_{j,t}^e \right\} \right]$$

$$(1 - \tau_t^e) r_t^k c_{j,t}^e - (1 - \tau_t^{fb}) b_{j,t}^e - \tilde{c}_{j,t}^e (1 - \delta) k_{j,t-1} + \frac{f^e_{l,t}}{\gamma_e} - \frac{f^e_{k,t}}{\gamma_e} + \frac{T_F}{\gamma_p+\gamma_i+\gamma_p} \right)$$

$$- \xi_{j,t}^e \left\{ (1 + r_l^{be}) b_{j,t}^e - m_t^e E_t \left\{ q_{t+1}^k \pi_{t+1} (1 - \delta) k_{j,t}^e \right\} \right\}.$$

The first order conditions of entrepreneurs with respect to $c_{j,t}^e$, $k_{j,t}^e$, and $b_{j,t}^e$ are:

$$\lambda_{j,t}^e (1 + \tau_c) - \frac{1 - a_c}{c_{j,t}^e - a_c c_{t-1}^e} = 0$$

$$q_{t}^{k} = (1 - \tau_k) r_t^{k} + \beta E_t \left\{ \frac{\lambda_{j,t+1}}{\lambda_{j,t}} \left[ q_{t+1}^{k} (1 - \delta) \right] \right\}$$

$$\lambda_{j,t}^e (1 - \tau_{fb}) - \tilde{c}_{j,t}^e (1 + r_l^{be}) - \beta E_t \left\{ \lambda_{j,t+1}^e \left( \frac{1 + r_l^{be}}{\pi_{t+1}} - \tau_{fb} \right) \right\} = 0.$$

Similarly to the case of households we consider a symmetric equilibrium where $c_{i}^e = c_{j,t}^e$, $k_{i}^e = k_{j,t}^e$, and $b_{i}^e = b_{j,t}^e$; therefore, the first order conditions associated with the
entrepreneurs’ problem become:

\[ \lambda^t_f (1 + \tau_c) - \frac{1 - a^e}{c^t_f - a^e c^t_{f-1}} = 0 \]

\[ q^k_f = (1 - \tau_k) r^k_f + \beta_c E_t \left\{ \frac{\lambda^t_{f+1}}{\lambda^t_f} \left[ q^k_{f+1}(1 - \delta) \right] \right\} + \gamma \]

\[ \lambda^t_f (1 - \tau_{fb}) - \xi^e_f (1 + r^b e) - \beta_c E_t \left\{ \lambda^t_{f+1} \left( \frac{1 + r^b e}{\pi_{f+1}} - \tau_{fb} \right) \right\} = 0. \]

Integrating across entrepreneurs the budget constraint of the entrepreneurs can be written as:

\[ (1 + \tau^t_f) c^t_f + \left( \frac{1 + r^b e}{\pi_{f-1}} - \tau^t_{fb} \right) b^t_{f-1} + q^k_f k^t_f = \]

\[ (1 - \tau^t_f) r^k f^t_f + (1 - \tau^t_{fb}) b^t_f + q^k_f (1 - \delta) k^t_{f-1} + \frac{\beta_c}{\gamma_c} + \frac{\beta_c}{\gamma_c} - \frac{\tau^t_f}{\gamma_p + \gamma_c + \gamma_p}, \]

and the binding borrowing constraint can be written as

\[ (1 + r^b e) b^t_f = m^t_c E_t \left\{ \frac{q^k_f}{\pi_{f+1}} (1 - \delta) k^t_f \right\}. \]

### 2.6 Intermediate good producers

There is a continuum of intermediate good producers with mass \( \gamma_x \). The \( j \)th intermediate good producer has access to a technology represented by a production function

\[ y^x_{j,t} = A_t \left( k_{j,t-1}^{e} u_{j,t} \right)^{\alpha} \left[ \left( \ell^{pp}_{j,t} \right)^{h_p} \left( \ell^{ii}_{j,t} \right)^{h_i} \left( \ell^{mm}_{j,t} \right)^{h_m} \right] ^{1 - \alpha} \left( \frac{K_{j,t-1}^{e}}{\gamma_x} \right)^{\alpha}. \]

where \( k_{j,t-1}^{e} \) is the capital rented by the firm, \( u_{j,t} \) controls how the firm utilizes capital, \( \ell^{pp}_{j,t} \) is the amount of packed patient labor input rented by the firm, \( \ell^{ii}_{j,t} \) is the amount of packed impatient labor input rented by the firm, \( \ell^{mm}_{j,t} \) is the amount of packed hand-to-mouth labor input rented by the firm, and \( K_{j,t-1}^{e} \) is the amount of public capital controlled by the government. \( A_t \) denotes an aggregate productivity shock with law of motion:

\[ \log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \sigma_A \varepsilon^A_t, \quad \varepsilon^A_t \sim \mathcal{N}(0, 1). \]

In addition, to the cost of the inputs required for production, the intermediate good producers face a capital utilization cost, \( \psi(u_{j,t}) \equiv \left[ \psi_{u_{1}}(u_{j,t} - 1) + \frac{\psi_{u_{2}}}{2}(u_{j,t} - 1)^2 \right] k_{j,t-1}^{e}. \]
and a fixed cost of production, \( \Phi_x \). The latter guarantees that the economic profits are roughly equal to zero in the steady state, to be consistent with the additional assumption of no entry and exit of intermediate good producers.

Intermediate good producers choose \( k^{ee}_{j,t-1}, \ell^{pp}_{j,t}, \ell^{ii}_{j,t}, \ell^{mm}_{j,t} \) and \( u_{j,t} \) for \( t = 0, 1, 2, \ldots \) to maximize

\[
E^t_0 \sum_{t=0}^{\infty} \beta^t \lambda^t \left\{ \frac{y^x_{j,t}}{x_t} - w^p_t \ell^{pp}_{j,t} - w^i_t \ell^{ii}_{j,t} - w^m_t \ell^{mm}_{j,t} - r^k_t k^{ee}_{j,t-1} - \psi(u_{j,t}) k^{ee}_{j,t-1} - \Phi_x \right\},
\]

where \( x_t \equiv \frac{P_t^x}{P_t^i} \), with \( P_t^x \) the nominal price of intermediate goods, subject to the production function for \( y^x_{j,t} \). Substituting the production function in the objective function we get:

\[
E^t_0 \sum_{t=0}^{\infty} \beta^t \lambda^t \left\{ \frac{A_t \left( k^{ee}_{j,t-1} u_{j,t} \right)^{\alpha}}{x_t} \left[ \left( \ell^{pp}_{j,t} \right)^{\mu_p} \left( \ell^{ii}_{j,t} \right)^{\mu_i} \left( \ell^{mm}_{j,t} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{P_t^x}{\gamma_e} \right)^{\alpha_g} \right. \\
- w^p_t \ell^{pp}_{j,t} - w^i_t \ell^{ii}_{j,t} - w^m_t \ell^{mm}_{j,t} - r^k_t k^{ee}_{j,t-1} - \left[ \psi(u_{j,t}) (u_{j,t} - 1) + \frac{\psi(u_{j,t})}{2} \right] k^{ee}_{j,t-1} - \Phi_x \}
\]

The first order conditions with respect to \( \ell^{pp}_{j,t}, \ell^{ii}_{j,t}, \ell^{mm}_{j,t}, k^{ee}_{j,t-1}, \) \( u_{j,t} \) yields

\[
\begin{align*}
\psi'(u_{j,t}) &= \frac{A_t}{x_t} \left( \frac{K_t^{g}}{\gamma_e} \right)^{\alpha_g} \left[ \left( \ell^{pp}_{j,t} \right)^{\mu_p} \left( \ell^{ii}_{j,t} \right)^{\mu_i} \left( \ell^{mm}_{j,t} \right)^{\mu_m} \right]^{1-\alpha} \\
&= \beta^t \lambda^t \left\{ \frac{\lambda^{ee}_{j,t+1}}{\lambda^{ee}_{j,t}} \left[ \psi'(u_{j,t+1}) u_{j,t+1} - \psi(u_{j,t+1}) \right] \right\}.
\end{align*}
\]

Notice that complete markets imply that \( \frac{\lambda^{ee}_{j,t+1}}{\lambda^{ee}_{j,t}} \) is constant, so we can drop the index \( j \) in the first order condition with respect to capital utilization (the last one). Hence, we have that:
• Firstly, after integrating out both sides of the first order conditions with respect to \( j \) we get

\[
\begin{align*}
\omega^p_t &= \frac{\mu^p (1 - \alpha)}{x_t} \frac{y^x_t}{\ell^pp_t} \\
\omega^i_t &= \frac{\mu^i (1 - \alpha)}{x_t} \frac{y^x_t}{\ell^ii_t} \\
\omega^m_t &= \frac{\mu^m (1 - \alpha)}{x_t} \frac{y^x_t}{\ell^mm_t} \\
\end{align*}
\]

where \( y^x_t = \left( \int_0^{y^x_t} \ell^ss_j d\ell \right) \) and \( \ell^ss_s = \left( \int_0^{\gamma_s} \ell^ss_j d\ell \right) \) for all \( s \in \{p, i, m\} \).

• Secondly, from the first order conditions it follows that the ratio of capital to labor are independent of \( j \)

\[
\begin{align*}
\frac{k^p_{j,t-1}}{\ell^pp_{j,t}} &= \frac{\alpha}{(1 - \alpha)} \frac{1}{\mu^p} \frac{1}{\psi'(u_t) u_t} = \frac{1}{\kappa_p,t} \\
\frac{k^i_{j,t-1}}{\ell^ii_{j,t}} &= \frac{\alpha}{(1 - \alpha)} \frac{1}{\mu^i} \frac{1}{\psi'(u_t) u_t} = \frac{1}{\kappa_i,t} \\
\frac{k^m_{j,t-1}}{\ell^mm_{j,t}} &= \frac{\alpha}{(1 - \alpha)} \frac{1}{\mu^m} \frac{1}{\psi'(u_t) u_t} = \frac{1}{\kappa_m,t} \\
\end{align*}
\]

These expression also imply that

\[
\begin{align*}
\frac{k^p_{t-1}}{\ell^pp_t} &= \frac{1}{\kappa_p,t} \\
\frac{k^i_{t-1}}{\ell^ii_t} &= \frac{1}{\kappa_i,t} \\
\frac{k^m_{t-1}}{\ell^mm_t} &= \frac{1}{\kappa_m,t} \\
\end{align*}
\]
where \( k^e_t = \left( \int_0^{\gamma_x} k^d_{j, t} \, dj \right) \).

Using the first order condition with respect to capital and the above expression yields

\[
\psi'(u_t) = \frac{A_t}{x_t} \left( \frac{K^S_{j-1}}{\gamma_x} \right)^{\alpha_g} \frac{\alpha}{\alpha_g} \left[ \frac{\left( k^e_{j, t-1} x_{p, t} \right)^{\mu_p} \left( k^e_{j, t-1} x_{i, t} \right)^{\mu_i} \left( k^e_{j, t-1} x_{m, t} \right)^{\mu_m}}{(k^e_{j, t-1} x_{t})^{1-\alpha}} \right]^{1-\alpha}
\]

\[
= \frac{A_t}{x_t} \left( \frac{K^S_{j-1}}{\gamma_x} \right)^{\alpha_g} \frac{1}{(k^e_{j, t-1})^{1-\alpha}} \left[ \frac{\left( k_{p, t} \right)^{\mu_p} \left( k_{i, t} \right)^{\mu_i} \left( k_{m, t} \right)^{\mu_m}}{(u_t)^{1-\alpha}} \right]^{1-\alpha}
\]

\[
= \frac{A_t}{x_t} \left( \frac{K^S_{j-1}}{\gamma_x} \right)^{\alpha_g} \frac{1}{(k^e_{j, t-1} x_{t})^{1-\alpha}} \left[ \frac{\left( \ell^p_{t} \right)^{\mu_p} \left( \ell^i_{t} \right)^{\mu_i} \left( \ell^m_{t} \right)^{\mu_m}}{(u_t)^{1-\alpha}} \right]^{1-\alpha}
\]

where the last line follows from the assumption of constant returns to scale on the production function, i.e. \( \alpha + (\mu_p + \mu_i + \mu_m)(1-\alpha) + \alpha_g = 1 \).

- Thirdly, we can solve for the ratio of private to public capital, which is independent of \( j \)

\[
\frac{k^e_{j, t-1}}{K^S_{j-1}} = \left( \frac{1}{\psi'(u_t)} \frac{A_t}{x_t} \frac{\left( k_{p, t} \right)^{\mu_p} \left( k_{i, t} \right)^{\mu_i} \left( k_{m, t} \right)^{\mu_m}}{(u_t)^{1-\alpha}} \right)^{\frac{1}{\alpha_g}} \left( \frac{1}{\gamma_x} \right)
\]

Hence \( \frac{k_{j, t-1}}{K_{j-1}^S} = \frac{k_{j-1}^e \gamma_x}{K_{j-1}^S} \).

Substituting these ratios into the production function yields
\[ y_{j,t}^X = A_t \left( \frac{k_{j,t-1}^{ee}}{\gamma_{j,t}} \right)^\alpha \left[ \left( \frac{k_{j,t-1}^{ee}}{k_{j,t-1}^{ee}} \right)^{\mu_p} \left( \frac{k_{j,t-1}^{ee}}{k_{j,t-1}^{ee}} \right)^{\mu_i} \left( \frac{k_{j,t-1}^{ee}}{k_{j,t-1}^{ee}} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K_{j,t-1}^S}{\gamma_x} \right)^{a_g} \]

\[ = A_t \left( \frac{k_{j,t-1}^{ee}}{k_{j,t-1}^{ee}} \right)^{\alpha} \left( \frac{k_{j,t-1}^{ee}}{k_{j,t-1}^{ee}} \right)^{(1-\alpha)(1-\alpha)(\mu_p+\mu_i+\mu_m)} \left[ \left( k_{j,t-1}^{ee} \right)^{\mu_p} \left( k_{j,t-1}^{ee} \right)^{\mu_i} \left( k_{j,t-1}^{ee} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K_{j,t-1}^S}{\gamma_x} \right)^{a_g} \]

\[ = k_{j,t-1}^{ee} A_t \left( k_{j,t-1}^{ee} \right)^{\alpha} \frac{1}{\left( k_{j,t-1}^{ee} \right)^{(1-\alpha)(1-\alpha)(1-\alpha)}} \left[ \left( k_{j,t-1}^{ee} \right)^{\mu_p} \left( k_{j,t-1}^{ee} \right)^{\mu_i} \left( k_{j,t-1}^{ee} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K_{j,t-1}^S}{\gamma_x} \right)^{a_g} \]

\[ = k_{j,t-1}^{ee} A_t \left( k_{j,t-1}^{ee} \right)^{\alpha} \frac{\left( k_{j,t-1}^{ee} \right)^{\alpha}}{\left( k_{j,t-1}^{ee} \right)^{(1-\alpha)(1-\alpha)(1-\alpha)}} \left[ \left( k_{j,t-1}^{ee} \right)^{\mu_p} \left( k_{j,t-1}^{ee} \right)^{\mu_i} \left( k_{j,t-1}^{ee} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K_{j,t-1}^S}{\gamma_x} \right)^{a_g} \]

This clearly implies that

\[ y_{j,t}^X = A_t \left( k_{j,t-1}^{ee} u_t \right)^\alpha \left[ \left( \frac{\epsilon_{p,t}^{pp}}{\epsilon_{t}^{pi}} \right)^{\mu_p} \left( \frac{\epsilon_{t}^{ei1}}{\epsilon_{t}^{pi}} \right)^{\mu_i} \left( \frac{\epsilon_{t}^{em}}{\epsilon_{t}^{m}} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K_{j,t-1}^S}{\gamma_x} \right)^{a_g} \]

Finally, the profits of the intermediate good producers are:

\[ \frac{I_{j,t}^X}{\gamma_x} = \frac{y_{j,t}^X}{x_t} - \omega_t^p \epsilon_{j,t}^{pp} - \omega_t^i \epsilon_{j,t}^{ei} - \omega_t^m \epsilon_{j,t}^{em} - r_t k_{j,t-1}^{ee} - \psi(u_{j,t})k_{j,t-1}^{ee} - \Phi_x \]

which, again, given that \( u_{j,t} = u_t \) for all \( j \in \gamma_x \) and integrating out both sides with respect to \( j \), yields:

\[ \frac{I_{j,t}^X}{\gamma_x} = \frac{v_{t}^X}{x_t} - \omega_t^p \epsilon_{t}^{pp} - \omega_t^i \epsilon_{t}^{ei} - \omega_t^m \epsilon_{t}^{em} - r_t k_{j,t-1}^{ee} - \psi(u_t)k_{j,t-1}^{ee} - \Phi_x \]

2.7 Capital producers

There is a continuum of capital goods producers, with mass \( \gamma_{k,t} \), who sell capital to entrepreneurs. At the beginning of each period, a capital good producer buys an amount \( i_{j,t} \) of final goods from final good packers at a price \( p_t^i = \frac{p_t}{\beta} \), and the stock of undepreciated capital \((1 - \delta)k_{j,t-1} \) from entrepreneurs at a price \( q_t^k \). Old capital can be converted one to one to new capital; however, the transformation of the final good is subject to quadratic
adjustment costs. Accordingly, the law of motion for capital, is

\[ k_{j,t} = (1 - \delta)k_{j,t-1} + \left[ 1 - \frac{\eta_i}{2} \left( \frac{i_{j,t} e_{i,t}}{i_{j,t-1}} - 1 \right) \right] i_{j,t}, \]

where \( e_i \) has the following law of motion:

\[ \log e_i = (1 - \rho e_i) \log e_{ss} + \rho e_i \log e_{t-1} + \sigma_k \epsilon_k \epsilon_i \sim \mathcal{N}(0, 1). \]

The new capital stock is then sold back to entrepreneurs at a price equal to \( q_{k,t} \). There is also a fixed cost of being a capital goods producer in order to guarantee that the profits are roughly zero around the steady state. Then, each capital good producer chooses \( k_{j,t} \) and \( i_{j,t} \) for \( t = 0, 1, 2, \ldots, +\infty \) to maximize:

\[ E_0 \sum_{t=0}^{+\infty} \beta^t \lambda^t \left\{ q_{k,t} [k_{j,t} - (1 - \delta)k_{j,t-1}] - p^t_{i,j,t} - \Phi_k \right\} \]

subject to the law of motion for capital. After substituting the constraint and taking the FOC with respect to \( i_{j,t} \) we get:

\[ q_{k,t} \left[ 1 - \frac{\eta_i}{2} \left( \frac{i_{j,t} e_{i,t}}{i_{j,t-1}} - 1 \right)^2 - \eta_i \left( \frac{i_{j,t} e_{i,t}}{i_{j,t-1}} - 1 \right) \left( \frac{e_i}{i_{j,t-1}} \right) i_{j,t} \right] + \]

\[ \beta_e E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} k_{t+1} e_{i,t+1} \eta_i \left( \frac{i_{t+1} e_{i,t+1}}{i_{t+1}} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right] \right\} = p^t_{i,t}. \]

By the assumption of complete markets \( \frac{\lambda_{t+1}}{\lambda_t} \) so that \( i_t = i_{j,t} \), and we can drop the subindex \( j \) in the previous equation

\[ q_{k,t} \left[ 1 - \frac{\eta_i}{2} \left( \frac{i_{t} e_{t}}{i_{t-1}} - 1 \right)^2 - \eta_i \left( \frac{i_{t} e_{t}}{i_{t-1}} - 1 \right) \left( \frac{e_i}{i_{t-1}} \right) i_t \right] + \]

\[ \beta_e E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} k_{t+1} e_{i} \eta_i \left( \frac{i_{t+1} e_{i,t+1}}{i_{t+1}} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right] \right\} = p^t_{i,t}, \]

and in the law of motion for capital

\[ k_t = (1 - \delta)k_{t-1} + \left[ 1 - \frac{\eta_i}{2} \left( \frac{i_{t} e_{t}}{i_{t-1}} - 1 \right)^2 \right] i_t. \]
Then, the profits of capital good producers are:

\[
\frac{\pi^k_t}{\gamma_k} = q^k_t \left\{ \left[ 1 - \frac{\eta_i}{2} \left( \frac{\epsilon^k_t}{i_{t-1}} - 1 \right) \right]^2 i_t \right\} - p^i_t i_t - \Phi_k.
\]

2.8 Home Goods Retailers

There is a continuum of final goods retailers, with mass \(\gamma\), operating in a monopolistically competitive market. Each good retailer buys an amount of the homogeneous intermediate good sold by intermediate goods producers, differentiates it and sells the resulting varieties at a mark-up over its marginal cost to "home-produced final goods packers" who in turn bundle the varieties together and sell the home-produced final homogeneous good to "final goods packers" that bundle home and imported production.

We assume that retail prices are indexed by a combination of past and steady state inflation with relative weights parameterized by \(\iota_p\). In addition, retailers are subject to quadratic price adjustment costs, where \(\eta_p\) controls the size of these costs.

Then, each retailer chooses the nominal price for its differentiated home produced good, \(p^H_{j,t}\), for \(t = 0, 1, 2, \ldots, +\infty\) to maximize:

\[
E_0 \sum_{t=0}^{+\infty} \beta_t p^\lambda_t p^H_t \left[ P^{H}_{y_j,t} - \frac{y_{x,t}}{x_t} \left( \frac{P^H_{j,t}}{P^H_{j,t-1}} - \left( \frac{\pi^H_t}{\pi_{ss,t}} \right)^{1-\iota_p} \right)^2 \pi_t \right]
\]

subject to

\[
y_{j,t} = y_{x,t}
\]

\[
y_{j,t} = \left( \frac{P^H_{j,t}}{p^H_t} \right)^{-\epsilon_{x,t}^y} \pi_t
\]

where \(\pi^H_t\) is the gross inflation of \(P^H_t\) and \(\epsilon_{x,t}^y\) is the elasticity of substitution which follows an AR(1) process with law of motion:

\[
\log \epsilon_{x,t}^y = (1 - \rho_{xy}) \log \epsilon_{ss}^y + \rho_{xy} \log \epsilon_{t-1}^y + \sigma_{xy} \epsilon_{t}^y \sim \mathcal{N}(0, 1).
\]

The demand faced by retailers is derived from the optimization problem solved by retail goods packers, left implicit.
The first order condition of the intermediate goods producer’s problem is:

\[
1 - \epsilon_t^{y} + \epsilon_t^{\pi} - \eta_p \pi_t^H \left( \pi_t^H - \left( \pi_{t-1}^H \right)^{i_p} \left( \pi_{ss}^H \right)^{1-i_p} \right) + \beta_p E_t \left\{ \frac{\lambda_{t+1}^p}{\lambda_t^p} \left[ \left( \frac{Y_{t+1}^H}{Y_t^H} \right)^2 \left( \frac{Y_{t+1}}{Y_t} \right) \eta_p \left( \pi_{t+1}^H - \left( \pi_t^H \right)^{i_p} \left( \pi_{ss}^H \right)^{1-i_p} \right) \right] \right\} = 0.
\]

We have omitted the subindices in the first order condition because of complete markets and the construction of a symmetric equilibrium as in the case of the patient households, which also imply that \( p_{j,t}^H = p_t^H \) and henceforth that

\[
Y_t = \left( \int_0^T \frac{Y_{j,t}^j}{Y_{j,t}^{H,j}} d\gamma \right)^{\frac{1-\epsilon_t^{y}}{1-\epsilon_t^{\pi}}}. 
\]

Finally, the retailers aggregate profits are:

\[
J_t^R = Y_t \left[ 1 - \frac{1}{x_t} - \frac{\eta_p}{2} \left( \pi_t^H - \left( \pi_{t-1}^H \right)^{i_p} \left( \pi_{ss}^H \right)^{1-i_p} \right)^2 \right]. 
\]

### 2.9 Banks

Banks play a key role in the model by intermediating financial transactions between the agents. In the model there is a continuum of them, with mass \( \gamma_b \), composed by three parts each one: a wholesale unit and two retail branches. The two retail branches are responsible for giving out differentiated loans and raising differentiated deposits from households and entrepreneurs, and are assumed to operate in monopolistically competitive markets. Specifically, each unit of deposits and loan contract bought by households and entrepreneurs are a CES basket of slightly differentiated products supplied by each branch \( j \), with packers intermediating the transactions and exploiting the market power derived from differentiation. The wholesale unit manages the capital position of the group, receives loans form abroad, and raises wholesale domestic loans and deposits in the interbank market.

**Banks: Wholesale branch**

There is a continuum of wholesale banks with mass \( \gamma_b \). The representative wholesale bank combine bank capital, \( k_{b,t} \), wholesale deposits, \( d_t^b \), and foreign borrowing, \( -\frac{B_t}{\gamma_b} \), in order to issue wholesale domestic loans, \( b_{j,t}^b \), everything expressed in terms of final consumption goods. The wholesale banks face costs on the wholesale activity related to
their capital position. Specifically, the banks pay a quadratic cost whenever the capital-to-assets ratio $k_{j,t}^{b}/b_{j,t}^{b}$ deviates from an exogenously given target. In addition, the banks must satisfy a balance sheet constraint linking equity, liabilities, and assets, i.e.

$$b_{j,t}^{b} = d_{j,t}^{b} - \frac{B_{t}^{*}}{\gamma_{b}} + k_{j,t}^{b},$$

Finally, bank capital, in nominal terms, $k_{j,t}^{b}$ evolves according to the following law of motion

$$k_{j,t}^{b} = \frac{(1 - \delta_{b})}{\varepsilon_{k_{j,t}^{b}}} k_{j,t-1}^{b} + \omega_{b}^{b} \pi_{j,t-1},$$

where $\varepsilon_{k_{j,t}^{b}}$ is a shock to the capital stock and $\pi_{j,t-1}$ represents the aggregate consolidated profits in nominal terms of the $j$-th bank holding in the economy from the deposit and loan contracts delivered at period $t$. Reexpressed in terms of $k_{j,t}^{b} = \frac{k_{j,t}^{b}}{\pi_{t}}$ and $j_{j,t}^{b} = \frac{j_{j,t}^{b}}{\pi_{t}}$, i.e., capital and profits in consumption good units, the latter expression becomes

$$p_{t} k_{j,t}^{b} = \frac{(1 - \delta_{b})}{\varepsilon_{k_{j,t}^{b}}} p_{t-1} k_{j,t-1}^{b} + \omega_{b} p_{t} j_{j,t-1},$$

or, equivalently,

$$\pi_{t} k_{j,t}^{b} = \frac{(1 - \delta_{b})}{\varepsilon_{k_{j,t}^{b}}} k_{j,t-1}^{b} + \omega_{b} \pi_{t} j_{j,t-1}^{b}.$$

Finally $\varepsilon_{k_{j,t}^{b}}$ follows the following law of motion,

$$\log \varepsilon_{k_{j,t}^{b}} = (1 - \rho_{ek_{j,t}^{b}}) \log \varepsilon_{k_{j,t}^{b}} + \rho_{ek_{j,t}^{b}} \log \varepsilon_{k_{j,t-1}^{b}} + \sigma_{ek_{j,t}^{b}} e_{k_{j,t}^{b}}$$

with $e_{k_{j,t}^{b}} \sim N(0, 1)$.

Given these definitions, the problem of the $j$-th wholesale bank is to choose the amount of wholesale loans, $b_{j,t}^{b}$, and wholesale deposits, $d_{j,t}^{b}$, for $t = 0, 1, 2, ..., +\infty$ in order to maximize the present value of intertemporal cash flows or, equivalently, by substituting the balance sheet equation into the latter expression the problem becomes one of unconstrained maximization of the following objective function,

$$\max_{b_{j,t}^{b}, d_{j,t}^{b}} \left( r_{t}^{b} k_{j,t}^{b} - r_{t} d_{j,t}^{b} + r_{t}^{*} B_{t}^{*} - \frac{\eta_{b}}{2} \left( \frac{k_{j,t}^{b}}{b_{j,t}^{b}} - v_{b} \right)^{2} \right) k_{j,t}^{b}$$

where $r_{t}^{b}$, $r_{t}$ and $r_{t}^{*}$ are the nominal gross interest rates for wholesale lending, domestic
fund raising and foreign borrowing, respectively, all of them taken as given by wholesale banks. The rate \( r_t \) is also the monetary policy rate what follows, in equilibrium, from the assumption that wholesale banks count on full-allotment availability of funds from the monetary authority at that rate. The first order condition displays the following results:

\[
(r_t^b - r_t) = -\eta_b \left( \frac{k_t^b}{b_t^b} - \nu_t \right) \left( \frac{k_t^b}{b_t^b} \right)^2
\]

\[
(1)
\]

\[
(r_t^b - r_t^*) = -\eta_b \left( \frac{k_t^b}{b_t^b} - \nu_t \right) \left( \frac{k_t^b}{b_t^b} \right)^2
\]

\[
(2)
\]

We can drop the subindex \( j \) from the first order conditions because we focus on a symmetric equilibrium where each wholesale unit \( j \) decides its optimal capital to loans ratio taking as given the capital to loans ratio of other banks. Accordingly, we can drop the subindices from the law of motion for bank capital yielding

\[
\pi_t k_t^b = \left( 1 - \delta_t \right) k_t^b_{t-1} + \omega_t \left( \frac{\pi_t f_t^b_{t-1}}{k_t^b} \right),
\]

and the balance sheet equation of each wholesale unit

\[
b_t^b = d_t^b - \frac{B_t^*}{Y_t} + k_t^b.
\]

where \( f_t^b \) is the aggregate of the consolidated profits for the \( \gamma_b \) bank holdings which populate the economy.

Following Schmitt-Grohé and Uribe (2003), to ensure stationarity of equilibrium we assume that banks pay a risk-premium that increases with the country’s net foreign asset position. Thus, we close the model by assuming that the rate \( r_t^* \) is equal to the exogenous interest rate \( r^{**} \) multiplied by a risk premium

\[
r_t^* = \phi_t r^{**}
\]

where the risk premium \( \phi_t \) increases with the deviation (in absolute value) of the ratio of net foreign asset position to output with respect to a constant \( b^* \)

\[
\phi_t = \exp \left( -\tilde{\phi} \left( \frac{B_t^*}{Y_t} - b^* \right) \theta_t^{pp} \right)
\]

\[
(4)
\]
and the shock $\theta_{r}^{p}$ obeys the following law of motion
\[
\log \theta_{r}^{p} = (1 - \rho_{\theta r p})\log \theta_{s s}^{p} + \rho_{\theta r p} \log \theta_{t - 1}^{p} + \sigma_{r}^{p} e_{r}^{p} \text{ with } e_{r}^{p} \sim \mathcal{N}(0, 1)
\]
Notice that equations (1), (2), and (3) imply that
\[r_{t} = \phi_{t} r_{t}^{*}\]

**Banks: Deposit-retailing branch**

There is a continuum of deposit-retailing banks with mass $\gamma_{b}$. Each retailing-deposit bank sells a differentiated type of deposit to deposit packers who bundle the deposits bought to all deposit-retailing banks according to a CES production function and sell the packed deposits to patient households. Finally, each deposit-retailing bank uses its resources to fund the wholesale unit at the monetary policy interest rate $r_{t}$.

The $j$th deposit-retailing bank chooses the path of the nominal gross interest rate paid by its type of deposit, $r_{d,j,t}$ for $t = 0, 1, 2, \ldots, +\infty$, in order to maximize:
\[
E_{0} \sum_{t=0}^{+\infty} \beta_{t}^{p} \left[ \frac{r_{d,j,t} - r_{d,j,t}^{pp}}{\epsilon_{d}^{l}} \left( \frac{r_{d,j,t}}{r_{d,j,t - 1}} - 1 \right) \right] r_{d,t}^{pp}
\]
subject to
\[
d_{b,j,t}^{p} = d_{j,t}^{pp},
\]
\[
d_{j,t}^{pp} = \left( \frac{r_{d,j,t}}{r_{d,t}} \right)^{-\epsilon_{d}^{l}} d_{j,t}^{pp},
\]

The last expression corresponds to the packers’ demand for the type of deposit supplied by the $j$th deposit-retailing bank\(^1\), where $\epsilon_{d}^{l}$ is the elasticity of substitution between types of deposit. In practice, we reparameterize this elasticity as $\epsilon_{d}^{l} \equiv \frac{\theta_{d}^{l}}{\theta_{d}^{l} - 1}$ with $\theta_{d}^{l}$, which is related to the flexible prices mark-down charged by deposit-retailing banks relative to the monetary policy rate, obeying the following law of motion:

\(^1\) The derivation of this demand is analogous to the one for the labor packers described above.
\[
\log \theta_t^d = (1 - \rho_{dd}) \log \theta_{ss}^d + \rho_d \log \theta_{t-1}^d + \sigma_d \varepsilon_t^d \text{ with } \varepsilon_t^d \sim \mathcal{N}(0,1).
\]

From the first order condition we get the following equilibrium condition for the representative deposit-retailing bank:

\[
1 + \frac{r_t}{r_t'} \left( \frac{\theta_t^d}{\theta_{t-1}^d} \right) - \left( \frac{\theta_t^d}{\theta_{t-1}^d} \right) + \eta_p \left( \frac{r_{t+1}}{r_t'} - 1 \right) \frac{r_{t+1}}{r_t'} \\
- \beta_p E_t \left\{ \frac{\lambda_{t+1}^p}{\lambda_t^p} \left[ \eta_p \left( \frac{r_{t+1}}{r_t'} - 1 \right) \left( \frac{r_{t+1}}{r_t'} \right)^2 \lambda_{t+1} - \lambda_t \right] \right\} = 0.
\]

where the index \( j \) has been dropped from the first order condition because of complete markets and the consideration of a symmetric equilibrium (see Rotemberg, 1982). Thus,

\[
d_t^p = d_t^{pp}.
\]

**Banks: Loan-retailing branch**

There is a continuum of loan-retailing banks with mass \( \gamma_b \). Each loan-retailing bank borrows wholesale loans from the wholesale unit at a rate \( r_t^b \), and sells a differentiated type of loan to loan packers who bundle the loans bought to all loan-retailing banks according to a CES production function and finally sell the packed loans to impatient households and entrepreneurs.

The \( j \)th loan-retailing bank chooses the nominal gross interest rates for its loans to impatient households, \( r^i_{jt} \), and entrepreneurs, \( r^e_{jt} \), for \( t = 0, 1, 2, ..., +\infty \), in order to maximize:

\[
E_0 \sum_{t=0}^{+\infty} \beta_t^p \lambda_t^p \left[ r^i_{jt} b^i_{jt} + r^e_{jt} b^e_{jt} + \theta_{ss}^p r_t^b \left( \frac{B^g}{\gamma_b} \right) - r_t^b b^i_{jt} - \frac{\eta_i}{2} \left( \frac{r^i_{jt}}{r^i_{jt-1}} - 1 \right) \right] 2 r^i_{jt} b^i_{jt} - \frac{\eta_e}{2} \left( \frac{r^e_{jt}}{r^e_{jt-1}} - 1 \right) 2 r^e_{jt} b^e_{jt} \\
\]

subject to

\[
b^i_{jt} = b^i_{jt} + b^e_{jt} + \frac{B^g}{\gamma_b},
\]
The last two expressions correspond to the loan packers’ demand for the type of loans supplied by the \( j \)th loan-retailing bank\(^2 \), where \( \varepsilon_{bi} \) and \( \varepsilon_{be} \) are the elasticities of substitution between types of loans for impatient households and for entrepreneurs, respectively. In practice, we reparameterize these elasticities as \( \varepsilon_{bs} \equiv \left( \frac{\theta_{bs}}{\theta_{bs-1}} \right) \) for \( s = i, e \) with \( \theta_{bs} \), which is related to the flexible prices mark-up charged by loan-retailing banks relative to the monetary policy rate, obeying the following law of motion:

\[
\log \theta_{bs} = (1 - \rho_{\theta_{bs}}) \log \theta_{ss}^d + \rho_d \log \theta_{t-1}^{bs} + \sigma_{bs} \varepsilon_{bs}^{bs} \text{ with } \varepsilon_{bs}^{bs} \sim N(0, 1).
\]

The first order conditions for this problem are:

\[
1 + \frac{r_t^b}{\lambda_t^b} \left( \frac{\theta_{bs}^b}{\theta_{bs-1}^b} \right) - \left( \frac{\theta_{bs}^b}{\theta_{bs-1}^b} \right) - \eta_{bi} \left( \frac{r_{bi}^b}{\lambda_{bi}^b} - 1 \right) \frac{r_{bi}^b}{\lambda_{bi}^b} + \beta_p E_t \left\{ \frac{\lambda_{p+1}}{\lambda_t^b} \left[ \eta_{bi} \left( \frac{r_{bi}^b}{\lambda_{bi}^b} - 1 \right) \left( \frac{r_{bi}^b}{\lambda_{bi}^b} \right)^2 \right] \right\} = 0
\]
\[
1 + \frac{r_t^e}{\lambda_t^e} \left( \frac{\theta_{bs}^e}{\theta_{bs-1}^e} \right) - \left( \frac{\theta_{bs}^e}{\theta_{bs-1}^e} \right) - \eta_{be} \left( \frac{r_{be}^e}{\lambda_{be}^e} - 1 \right) \frac{r_{be}^e}{\lambda_{be}^e} + \beta_p E_t \left\{ \frac{\lambda_{p+1}}{\lambda_t^e} \left[ \eta_{be} \left( \frac{r_{be}^e}{\lambda_{be}^e} - 1 \right) \left( \frac{r_{be}^e}{\lambda_{be}^e} \right)^2 \right] \right\} = 0
\]

where again we dropped the subindex \( j \) from the individual first order condition because of the reasons mentioned above. It allows also to write,

\[
b_t^b = b_t^{bi} + b_t^{be} + \frac{B_t^g}{\gamma_b}.
\]

\(^2\) The derivation of these demands, as in the case of the deposit packers, is analogous to the one for the labor packers described above.
Bank holding’s profits

The consolidated profit of the $j$-th bank holding in consumption good units is given by

$$\pi^b_{j,t} = r^b_{j,t} + r^c_{j,t} + \theta^b_{j,t} r^b_{j,t} - r^d_{j,t} d^m_{j,t} + r^f_{j,t} b^c_{j,t} - \eta_d \left( \frac{k^b_{j,t}}{k^b_{j,t-1}} - \nu_b \right)^2 k^b_{j,t} - \frac{\eta_d}{2} \left( \frac{r^d_{j,t}}{r^d_{j,t-1}} - 1 \right)^2 r^d_{j,t} d^m_{j,t} - \frac{\eta_b}{2} \left( \frac{r^f_{j,t}}{r^f_{j,t-1}} - 1 \right)^2 r^f_{j,t} b^c_{j,t}.$$  

2.10 External Sector

We consider a world of two asymmetric countries in which the home country is small relative to the other (the rest of the world), whose equilibrium is in the limit taken as exogenous (see Monacelli, 2003 or Galí and Monacelli, 2005). As the price numeraire is before tax CPI, we define relative prices below in lower case.

Imports

We assume that there is a continuum of final consumption good packers with mass $\gamma_c$ that buy domestic $c^h_{j,t}$ and foreign $c^f_{j,t}$ consumption goods, pack them and sell the bundle to households and entrepreneurs. The aggregation technology is expressed by the following CES composite baskets of home and foreign produced goods.

$$c^c_{j,t} = \left( 1 - \omega_c \right)^{\frac{1}{\sigma_c}} \left( c^h_{j,t} \right)^{\frac{\sigma_c-1}{\sigma_c}} + \omega_c \left( c^f_{j,t} \right)^{\frac{\sigma_c-1}{\sigma_c}} \left( \frac{\sigma_c}{\sigma_c-1} \right)$$  

Similarly, we consider the existence of a continuum of final investment packers with mass $\gamma_l$ that buy domestic $i^h_{j,t}$ and foreign $i^f_{j,t}$ investment goods, pack them and sell the bundle to capital producers. The technology is given by

$$i^i_{j,t} = \left( 1 - \omega_i \right)^{\frac{1}{\sigma_i}} \left( i^h_{j,t} \right)^{\frac{\sigma_i-1}{\sigma_i}} + \omega_i \left( i^f_{j,t} \right)^{\frac{\sigma_i-1}{\sigma_i}} \left( \frac{\sigma_i}{\sigma_i-1} \right)$$

where $\sigma_c$ and $\sigma_i$ are the consumption and investment elasticities of substitution between domestic and foreign goods and $\omega_c$, $\omega_i$ is inversely related with the degree of home bias and, therefore, of openness.

Each period, the consumption packer chooses $c^h_{j,t}$ and $c^f_{j,t}$ to minimize production costs subject to the technological constraint given by (5). The Lagrangian of this problem
can be written as:

\[
\min_{c^h_{jt}, c^f_{jt}} \left\{ \left( P^H_t c^h_{jt} + P^M_t c^f_{jt} \right) + P_t \left[ c^h_{jt} - \left( 1 - \omega_c \right)^{\frac{\sigma_c}{\sigma_c - 1}} \left( c^h_{jt} \right)^{\frac{\sigma_c - 1}{\sigma_c}} + \omega_c \left( c^f_{jt} \right)^{\frac{\sigma_c - 1}{\sigma_c}} \right] \right\}
\]

(7)

where \( P^H_t \) and \( P^M_t \) are respectively the prices of home and foreign produced goods (expressed in domestic currency and including import tariffs). Note that \( P_t \) represents both the consumer price index (CPI) before the consumption tax and the shadow cost of production of a final good faced by the aggregator.

The optimal allocation of aggregate consumption between domestic and foreign goods, \( c^h_{jt} \) and \( c^f_{jt} \), satisfies the following conditions:

\[
c^h_{jt} = (1 - \omega_c) \left( p^H_t \right)^{-\sigma_c} c^c_{jt}
\]

(8)

\[
c^f_{jt} = \omega_c \left( p^M_t \right)^{-\sigma_c} c^c_{jt}
\]

(9)

Similarly, the investment distributor chooses \( i^h_{jt} \) and \( i^f_{jt} \) to minimize production costs subject to the technological constraint given by (6) by minimizing the Lagrangian

\[
\min_{i^h_{jt}, i^f_{jt}} \left\{ \left( P^H_t i^h_{jt} + P^M_t i^f_{jt} \right) + P_I \left[ i^h_{jt} - \left( 1 - \omega_i \right)^{\frac{\sigma_i}{\sigma_i - 1}} \left( i^h_{jt} \right)^{\frac{\sigma_i - 1}{\sigma_i}} + \omega_i \left( i^f_{jt} \right)^{\frac{\sigma_i - 1}{\sigma_i}} \right] \right\}
\]

(10)

from which the demand for home and foreign investment goods are obtained

\[
i^h_{jt} = (1 - \omega_i) \left( \frac{p^H_I}{p^I_t} \right)^{-\sigma_i} i^\varepsilon_{jt}
\]

(11)

\[
i^f_{jt} = \omega_i \left( \frac{p^M_I}{p^I_t} \right)^{-\sigma_i} i^\varepsilon_{jt}
\]

(12)

By assuming a symmetric equilibrium we can drop the subindex \( j \) from previous equations to get
In order to obtain the expression behind the CPI, the demands for home and foreign consumption goods need to be incorporated into the cost of producing final consumption goods ($P_{tc} = P_{th} c_h + P_{tm} c_f$). Bearing in mind that the unitary production cost for the distributor is equal to the price of producing one unit of the packed good, the consumption and investment price index (before consumption and investment tax/subsidy) can be expressed as a function of the domestic and import deflators\(^3\):

\[
P_t = \left( (1 - \omega_c) \left( \frac{P_{th}}{P_{ht}} \right)^{1-\sigma_c} + \omega_c \left( \frac{P_{tm}}{P_{mt}} \right)^{1-\sigma_c} \right)^{\frac{1}{1-\sigma_c}}
\]

\[
P_{t}^{f} = \left( (1 - \omega_i) \left( \frac{P_{th}}{P_{ht}} \right)^{1-\sigma_i} + \omega_i \left( \frac{P_{tm}}{P_{mt}} \right)^{1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}}
\]

Let’s define CPI gross inflation as $\pi_t = \frac{P_t}{P_{t-1}}$ and the after consumption tax inflation as $\pi_t' = \frac{P_t}{P_{t-1}} \frac{1+\tau_t}{1+\tau_{t-1}}$. This will be the inflation monitored by the Central Bank. The two previous equations can be written in relative terms as

\[
1 = \left( (1 - \omega_c) \left( \frac{P_{th}}{P_{ht}} \right)^{1-\sigma_c} + \omega_c \left( \frac{P_{tm}}{P_{mt}} \right)^{1-\sigma_c} \right)^{\frac{1}{1-\sigma_c}}
\]

\(^3\) Under the assumption that $\omega_c = \omega_i$ and $\sigma_c = \sigma_i$, the consumption and investment price indexes are the same (this is the approach in Farhi et al, 2011 or Stähler and Thomas, 2012).
\[ p^I_t = \left((1 - \omega_i) \left( p^H_t \right)^{1-\sigma_i} + \omega_i \left( p^M_t \right)^{1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}} \]  

(20)

Given the small open economy assumption, the price of imports in domestic currency is defined as:

\[ p^M_t = e_{rt}(1 + \tau^m_t) \]  

(21)

where \( e_{rt} \) is the real exchange rate (and \( E_{rt} \) the nominal exchange rate), i.e., \( e_{rt} = \frac{E_{rt}}{P_t} \frac{P_t}{P_t^*} \), \( \tau^m_t \) represents the import tariff\(^4\) and \( P_t^* \) stands for the exogenous world price index.

Some aggregate definitions follow from the previous equations

\[ C_t = \gamma_c c^c_t \]  

(22)

\[ C_{ht} = \gamma_c c^h_t \]  

(23)

\[ I_t = \gamma_z i^z_t \]  

(24)

\[ I_{ht} = \gamma_z i^h_t \]  

(25)

and total imports are

\[ IM_t = \gamma_c c^c_t + \gamma_z i^z_t = C_{ft} + I_{ft} \]  

(26)

Therefore, the following equalities hold in aggregate

\[ C_t = \gamma_c c^c_t = p^H_t \gamma_c c^h_t + p^M_t \gamma_c c^f_t = \gamma_p c^p_t + \gamma_i c^i_t + \gamma_c c^c_t + \gamma_m c^m_t \]  

(27)

\[ I_t = \gamma_z i^z_t = \frac{p^H_t}{p^I_t} \gamma_z i^h_t + \frac{p^M_t}{p^I_t} \gamma_z i^f_t = \gamma_k i^k_t \]  

(28)

\(^4\) In a full monetary union the tariff rate is zero.
Exports

Export prices set by domestic firms could deviate from competitors’ prices in foreign markets (there is some degree of imperfect exchange rate pass through). To make this assumption operational, we can think of a fraction \((1 - ptm)\) of retailing firms whose prices at home and abroad differ. The remaining fraction of firms, \(ptm\), trade goods freely setting a unified price across countries (i.e., the law of one price holds). Thus, the export price deflator is defined as

\[
p_{t}^{ lex} = (1 - \tau_{t}^{x})(p_{t}^{H(1 - ptm)})^{ptm} \tag{29}
\]

where \(p_{t}^{ lex}\) is the export price deflator, \(\tau_{t}^{x}\) is an export subsidy and the parameter \(ptm\) determines the degree of pass through.

A set of analogous optimality conditions characterize the solution to the consumer’s problem in the world economy, where a continuum Dixit-Stiglitz aggregators with mass \(\gamma^{*}\) faces a problem identical to the one outlined above for consumption and investment\(^5\).

The symetric conditions for foreign consumption of home-produced goods are therefore

\[
c_{t}^{*f} = \omega_{c}^{*} \left( \frac{p_{t}^{ lex}}{er_{t}} \right)^{-\sigma_{c}^{*}} c_{t}^{*} \tag{30}
\]

\[
i_{t}^{*f} = \omega_{i}^{*} \left( \frac{p_{t}^{ lex}}{er_{t}} \right)^{-\sigma_{i}^{*}} i_{t}^{*} \tag{31}
\]

where \(c_{t}^{*}\) and \(i_{t}^{*}\) represent the (exogenous) consumption and investment demand in the world economy. Therefore, exports of the home economy \(ex_{t} = c_{t}^{*f} + i_{t}^{*f}\) can be written as

\[
ex_{t} = \omega_{c}^{*} \left( \frac{p_{t}^{ lex}}{er_{t}} \right)^{-\sigma_{c}^{*}} (c_{t}^{*} + i_{t}^{*}) \tag{32}
\]

Plugging (29) into (32) yields the exports demand for a small open economy

\[
ex_{t} = \omega_{c}^{*} \left( (1 - \tau_{t}^{x}) \left( \frac{p_{t}^{H}}{er_{t}} \right)^{(1 - ptm)} \right)^{-\sigma_{c}^{*}} (c_{t}^{*} + i_{t}^{*}) \tag{33}
\]

Note that with full pricing to market \((ptm = 0)\), \(p_{t}^{ lex} = (1 - \tau_{t}^{x})p_{t}^{H}\) and expression (32)

\(^{5}\) For simplicity, we assume that the CES basket is the same for consumption and investment.
simplifies to

\[ ex_t = \omega^*_c \left( 1 - \tau^*_t \right) \left( \frac{p^H_t}{er_t} \right)^{-\sigma^*_c} (c^*_t + i^*_t) \] (34)

Conversely, if the law of one price holds for all consumption and investment goods, then \( ptm = 1 \), \( p^EX_t = (1 - \tau^*_t)er_t \) and expression (32) simplifies to

\[ ex_t = \omega^*_c(1 - \tau^*_t)^{-\sigma^*_c}(c^*_t + i^*_t) \] (35)

Thus, if the law of one price holds, exports are a sole function of total aggregate consumption and investment from abroad. Under full pricing to market \( (ptm = 0) \), exports are also a function of relative prices with elasticity \( \sigma^*_c \). Under the more general case of partial pricing-to-market \( (0 < ptm < 1) \), the price elasticity of exports is given by \( (1 - ptm) \sigma^*_c \).

Finally, we can define aggregate exports as

\[ EX_t = \gamma^* ex_t \] (36)

**Accumulation of foreign assets**

The net foreign asset position \( B^*_t \) evolves according to the following expression (denominated in the home currency)

\[ B^*_t = \frac{(1 + r^*_t)^L}{\pi} B^*_t - 1 + \left[ p^EX_t^* ex_t - p^M_t \left( \gamma^*_c c^f_t + \gamma^*_z i^f_t \right) \right] \] (37)

where a negative/positive sign for \( B^*_t \) implies a borrowing/lending position for the domestic economy with respect to the rest of the world and \( r^*_t \) stands for the interest rate paid/received for borrowing/lending abroad. Also, trade balance \( TB_t \) is defined as

\[ TB_t = p^EX_t^* ex_t - p^M_t \left( \gamma^*_c c^f_t + \gamma^*_z i^f_t \right) \] (38)

### 2.11 Monetary authority

The home economy belongs to a monetary union (say EMU), and monetary policy is managed by the central bank (say ECB) through the following Taylor rule that sets the nominal area-wide reference interest rate allowing for some smoothness of the interest
rate response to the inflation and output

\[(1 + r_t) = (1 + r_{ss})(1 - \phi_r)(1 + r_{t-1})^{\phi_r} \left( \frac{\pi^\text{emu}_{t}}{\pi^\text{emu}_{t-1}} \right)_{1 - \phi_r} \phi_r (1 - \phi_r) (1 + e_t^r) \]

where \(\pi^\text{emu}_t\) is the EMU inflation as measured in terms of the consumption price deflator and \(\frac{y^\text{emu}_{t}}{y^\text{emu}_{t-1}}\) measures the gross rate of growth of EMU output. There is also some inertia in nominal interest rate setting.

The home economy contributes to EMU inflation and output growth according to its economic size in the euro zone, \(\omega_{Sp}\):

\[\pi^\text{emu}_t = (1 - \omega_{Sp}) \left( \frac{\pi^\text{emu}_{t-1}}{\pi^\text{emu}_{t-1}} e^\pi_t \right) + \omega_{Sp} \pi^\text{emu}_{t-1} \quad (39)\]

\[\frac{y^\text{emu}_{t}}{y^\text{emu}_{t-1}} = (1 - \omega_{Sp}) \left( \frac{y^\text{emu}_{t-1}}{y^\text{emu}_{t-1}} e^y_{t} \right) + \omega_{Sp} \frac{y_{t}}{y_{t-1}} \quad (40)\]

where \(\pi^\text{emu}_{t}\) and \(\frac{y^\text{emu}_{t}}{y^\text{emu}_{t-1}}\) are average inflation and output growth in the rest of the Eurozone that are subject to shocks modeled according to the following law of motion

\[\log e^\pi_t = (1 - \rho_{\theta p}) \log e^\pi_{t-1} + \rho_{\theta p} \log e^\pi_{t-1} + \sigma_{\theta p} e^\pi_t + e^\pi_t \sim \mathcal{N}(0, 1)\]

\[\log e^y_t = (1 - \rho_{\theta p}) \log e^y_{t-1} + \rho_{\theta p} \log e^y_{t-1} + \sigma_{\theta p} e^y_t + e^y_t \sim \mathcal{N}(0, 1)\]

However, instead of a Taylor rule we use the following condition to determine the domestic interest rate

\[r_t = \phi_r r^*_t \quad (41)\]

where \(\phi_r\) is a parameter that relates the rate of borrowing abroad and the domestic interest rate.

The real exchange rate is given by the ratio of relative prices between the domestic economy and the remaining EMU members, so real appreciation/depreciation developments are driven by the inflation differential of the domestic economy vis-à-vis the euro area:

\[\frac{er_{t+1}}{er_{t}} = \frac{\pi^\text{emu}_{t+1}}{\pi^\text{emu}_{t+1}} \quad (42)\]
2.12 Fiscal authority’s budget Constraint

There is also a fiscal authority with a flow of expenses determined by consumption, investment on public capital, and interest plus principal borrowed during the previous period. The fiscal authority collects revenues with new debt, lump-sum taxes, and distortionary taxation on consumption, housing services, labor income, loans, and deposits.

\[
C_t^g + I_t^g + \left( \frac{1 + \theta_b t^{gb}_{t-1}}{\alpha_t} \right) B_{t-1}^{g} = B_t^{g} + T_t^{g} + \tau_{1}^{c} \left[ \gamma_p c_t^p + \gamma_i c_t^i + \gamma_c c_t^c + \gamma_m c_t^m \right] + \frac{\tau^{m}}{1 + \tau^{m}} p_t^M M_t - \frac{\tau^{f}}{1 - \tau^{f}} p_t^E EX_t + \tau^{h} q_{ht}^{h} \left( h_t^p - h_{t-1}^p \right) + \tau^{c} \gamma_i (h_t^i - h_{t-1}^i) + \tau^{w} \left[ w_t^p \gamma_p \ell_t^p + w_t^i \gamma_i \ell_t^i + w_t^m \gamma_m \ell_t^m \right] + \tau^{f} r_{1}^{f} K_t + \tau^{h} \gamma_i (b_t^i - b_{t-1}^i) + \gamma_c (b_t^c - b_{t-1}^c) + \tau^{d} \gamma_p (d_t^p - d_{t-1}^p) + \tau^{f} \left( \frac{1}{1 - \tau^{f}} \right) \gamma_p d_t^{p-1}.
\]

Tax rates are determined according to the following fiscal policy rule

\[
\tau_{1}^{s} = \tau_{s} \text{ for } s = c, h, w, d, f d, f b, r, m, x.
\]

\[
\psi_{1}^{bg} = \left( \frac{B_t^{g}}{\gamma y_t} \right)
\]

\[
\psi_{1}^{cg} = \left( \frac{C_t^{g}}{\gamma y_{ss}} \right)
\]

\[
\psi_{1}^{ig} = \left( \frac{T_t^{g}}{\gamma y_{ss}} \right)
\]

\[
\psi_{1}^{bg} = (1 - \rho_{bg}) \psi_{ss}^{bg} + \rho_{bg} \psi_{1}^{bg} + \sigma_{bg} c_t^{bg}
\]

\[
\psi_{1}^{ig} = (1 - \rho_{bg}) \psi_{ss}^{ig} + \rho_{bg} \psi_{1}^{ig} + \sigma_{bg} c_t^{ig}
\]

\[
\psi_{1}^{ig} = \psi_{1}^{ig} + \rho_{tgb1} (\psi_{1}^{bg} - \psi_{ss}^{bg}) + \rho_{tgb2} (\psi_{1}^{bg} - \psi_{1}^{bg})
\]

Finally, public capital evolves according to the law of motion

\[
K_t^{g} = (1 - \delta_{g}) K_{t-1}^{g} + I_t^{g}.
\]
2.13 Aggregation and market clearing in equilibrium

Factors of production markets

The supply of labor by each category of household equals the corresponding demand for it from intermediate good producers, i.e.,

\[ \int_0^{\gamma p} \ell_{j,t}^p \, dj = \int_0^{\gamma s} \ell_{j,t}^{pp} \, dj \Rightarrow \gamma_p \ell_t^p = \gamma_s \ell_{j,t}^{pp} \]

\[ \int_0^{\gamma i} \ell_{j,t}^i \, dj = \int_0^{\gamma s} \ell_{j,t}^{ii} \, dj \Rightarrow \gamma_i \ell_t^i = \gamma_x \ell_{j,t}^{ii} \]

\[ \int_0^{\gamma m} \ell_{j,t}^m \, dj = \int_0^{\gamma s} \ell_{j,t}^{mm} \, dj \Rightarrow \gamma_m \ell_t^m = \gamma_x \ell_{j,t}^{mm} \]

The supply of physical capital units by capital producers equals the corresponding demand by entrepreneurs, while the supply of physical capital rental services by the latter equals the demand of these services by intermediate good producers, i.e.,

\[ \int_0^{\gamma c} k_{j,t}^c \, dj = \int_0^{\gamma s} k_{j,t}^c \, dj \Rightarrow \gamma_c k_t^c = \gamma_k k_t \]

\[ \int_0^{\gamma c} k_{j,t}^{cc} \, dj = \int_0^{\gamma s} k_{j,t}^{cc} \, dj \Rightarrow \gamma_x k_t^{cc} = \gamma_x k_t^c \]

Housing market

The demand for houses by households equals a perfectly inelastic supply of houses,

\[ \int_0^{\gamma h} h_{j,t}^h \, dj + \int_0^{\gamma h} h_{j,t}^{hp} \, dj = H_t \Rightarrow \gamma_p h_t^h + \gamma_i h_t^i = H_t \]

Intermediate goods

The demand of intermediate goods by final good producers or retailers equals the supply of them by intermediate good producers,
\[
\int_0^\gamma y_{ij,t}^x \, dj = \int_0^\gamma y_{ij,t}^{xx} \, dj \Rightarrow \gamma_j y_t^x = \gamma y_t
\]

where the last equality follows from the production function for final goods, \( y_{ij,t} = y_{ij,t}^{xx} \).

**Banking services market**

The loan demand by impatient households and entrepreneurs equals the corresponding supply by loan-retailing-banks,

\[
\int_0^\gamma b_{ij,t}^i \, dj = \int_0^\gamma b_{ij,t}^{ii} \, dj \Rightarrow \gamma_i b_t^i = \gamma_{ib_t}^i,
\]

\[
\int_0^\gamma b_{ij,t}^e \, dj = \int_0^\gamma b_{ij,t}^{ee} \, dj \Rightarrow \gamma_e b_t^e = \gamma_{eb_t}^e,
\]

and the deposit demand by patient households equals the deposit supply by deposit-retailing banks,

\[
\int_0^\gamma d_{ij,t}^p \, dj = \int_0^\gamma d_{ij,t}^{pp} \, dj \Rightarrow \gamma_p d_t^p = \gamma_{dp_t}^p.
\]

**Final goods**

The demand of final goods by households, entrepreneurs and capital producers equals the supply of them by final goods packers,

\[
\int_0^\gamma c_t^c \, dj = \gamma_c c_t^c = \gamma_p c_t^p + \gamma_i c_t^i + \gamma_e c_t^e + \gamma_m c_t^m
\]

\[
\int_0^\gamma \tilde{t}_{ij} \, dj = \gamma_z \tilde{t}_t = \gamma_{zj} \tilde{t}_t
\]

By aggregating the budget constraints of households and plugging the market clearing conditions it can be derived the following expression for the effective aggregate demand for final goods in equilibrium,
\[ p_t^H Y_t = C_t + p_t^I I_t + p_t^H C_t^g + p_t^H I_t^g + p_t^EXEX_t - p_t^IMt \]

\[ + \left[ \psi_{u_t} (u_t - 1) + \frac{\psi_{u_t}}{2} (u_t - 1)^2 \right] K_{t-1} + \delta b \frac{K_{t-1}^{b_t}}{\pi_t} + \frac{\eta_p}{2} \left( \pi_t - \pi_{t-1}^{p_t} \pi^{1-p_t} \right)^2 Y_t \]

\[ + \frac{1}{\pi_t} \left[ \frac{\eta_k}{2} \left( \frac{r_{t-1}^p}{r_{t-2}^p} - 1 \right)^2 r_{t-1}^p D_{t-1} + \frac{\eta_k}{2} \left( \frac{r_{t-1}^i}{r_{t-2}^i} - 1 \right)^2 r_{t-1}^i B_{t-1} + \frac{\eta_k}{2} \left( \frac{r_{t-1}^e}{r_{t-2}^e} - 1 \right)^2 r_{t-1}^e B_{t-1} \right] \]

\[ + \frac{\eta_k}{2} \left( \frac{k_{t-1}^b}{\pi_t} - v_t \right) \frac{K_{t-1}^{b_t}}{\pi_t} + \gamma \frac{\eta_{iw}}{2} \left( \pi_t^{iw} - \pi_{t-1}^{iw} \pi^{1-iw} \right)^2 w_t^p + \gamma \frac{\eta_{iw}}{2} \left( \pi_t^{iw} - \pi_{t-1}^{iw} \pi^{1-iw} \right)^2 w_t^i \]

\[ + \frac{\gamma_{m} \eta_{mw}}{2} \left( \pi_t^{mw} - \pi_{t-1}^{mw} \pi^{1-iw} \right)^2 w_t^m, \]

where

\[ Y_t = \gamma y_t, \]

\[ C_t = \gamma p c_t^p + \gamma i c_t^i + \gamma m c_t^m + \gamma e c_t^e = \gamma p_t^H C_t + \gamma p_t^M C_t, \]

\[ I_t = \gamma k_t^i = \frac{p_t^M}{p_t^H} I_t + \frac{p_t^M}{p_t^H} I_t, \]

\[ K_{t-1}^{b_t} = \gamma k_{t-1}^{b_t}, \]

\[ K_{t-1}^{b_t} = \gamma k_{t-1}^{b_t}, \]

\[ D_t = \gamma p_d_t^d, \]

\[ B_t^i = \gamma b_t^i, \]

\[ B_t^e = \gamma b_t^e, \]

\[ B_t = B_t^e + B_t^i + B_t^g. \]

and, given that, by the market clearing condition, \( \gamma y_t = \gamma x y_t^{x} \) we can derive the following expression for the effective aggregate supply of final goods in equilibrium.

\[ Y_t = \gamma x A_t \left( k_{t-1}^{ce} u_t \right)^{\alpha} \left[ \left( \ell_{t}^{pp} \right)^{\mu_p} \left( \ell_{t}^{ii} \right)^{\mu_i} \left( \ell_{t}^{mm} \right)^{\mu_m} \right]^{\frac{1}{1-\alpha}} \left( K_{t-1}^{g} \right)^{\alpha g}. \]

Finally, GDP, \( Y_t \), can be defined as
\[ p_i^{H}Y_i^1 = C_t + p_i^I I_t + p_i^H C_t^8 + p_i^H I_t^8 + p_i^{EX}EX_t - p_i^{M}IM_t = \]
\[ = p_i^H C_{ht} + p_i^H I_{ht} + p_i^H C_t^8 + p_i^H I_t^8 + p_i^{EX}EX_t \]

### 2.14 Calibration

The table 1a shows the calibrated parameters that our model share with Gerali et.al’s model. We follows Gerali et.al’s calibration values and conventions with a few exceptions:

- We update the values of the interest rates spreads having into account that in our model household loans comprehend total banking loan and not only housing loans as in Gerali et.al, and our updating of the data. We use averages for the sub-sample, 1997Q1-2006Q4 to avoid the influence of the atypical pre- and post-financial-crisis observations.

- For giving room to hand-to-mouth households’ labor in the production function we re-weighted the share parameters: \( \mu_p \) and \( \mu_i \), corresponding to \( \mu \) and \( 1 - \mu \) in Gerali et.al. In particular, we follows Gali, López-Salido and Valles (2004) at fixing at 50% the share of financially unconstrained households’ labor, \( \mu_p \), and other 50% the share of financially constrained household’s labor by fixing at 25% the individual share of impatient households, \( \mu_i \), and hand-to-mouth households, \( \mu_p \) (see Table 1b).

- We increase the share of private physical capital in the production function to 0.37, a middle point between the value of Gerali et.al and that of the QUESTIII model for europe (see Ratto et.al., 2008).

The Table 1b shows the calibrated subset of the new parameters –i.e, those absent from Gerali et.al– associated to the hand-to-mouth households and fiscal policy. Their values were fixed according to the following criteria:

- The discount factor of the hand-to-mouth households and the share of their supply of labor in the production function are equated to that of impatient households.

- The population mass of the hand-to-mouth households, \( \gamma_{m} \), is estimated.

- The value of the share in the production of public physical capital, \( \alpha_g \), and its depreciation rate, \( \delta_g \), are taken from the QUESTIII model for Europe (see Ratto et.al., 2008).

- The tax rates on financial transactions (\( \tau_{fb} \), \( \tau_{fd} \)) and on deposits’ interest yield (\( \tau_d \)) are fixed in zero, while the value for the taxes on consumption (\( \tau_c \)), wages (\( \tau_w \)) and capital (\( \tau_k \)) corresponds to the average effective rates estimated in “Taxation trends in the European Union”, European Commission, 2012 edition.

- The values for the parameters of the public consumption’s and public investment’s policy rules and the standard deviation of the corresponding shocks were computed by least squares regression using the data sub-sample for 1997Q1-2006Q4.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
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<td>$\beta_p$</td>
<td>Patient households’ discount factor</td>
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</tr>
<tr>
<td>$\beta_i$</td>
<td>Impatient households’ discount factor</td>
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<td>$\beta_e$</td>
<td>Entrepreneurs’ discount factor</td>
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<td>$\phi$</td>
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Table 1a: Calibrated parameters: Shared with Gerali’s et-al

- The values for the parameters of lump-sum-taxes policy are taken from Boscá, Domenech et.al. (2010) and are intended to guarantee the stability of public debt, with the only difference that our autoregressive parameter is fixed at 0.99 instead of 1.0.

2.15 Estimation
We estimate exactly the same parameters than Gerali et.al plus the population mass of the hand-to-mouse households, $\gamma_m$, and use the same prior distribution, adding just an independent gamma prior for the latter parameter and performing a mean preserving spreading (increase of variances) of the distribution for the parameters of the Taylor’s rule, for giving room to capture the break associated to the financial crisis and the zero lower bound constraint on the policy interest rate. Specifically, ...
Table 1b: Calibrated parameters: Hand-to-Mouth households & Fiscal policy

(To be concluded)

3. Conclusions
(To be completed)
Appendix 1: Equilibrium conditions in summary

1. Markets, agents and first order conditions

1.1 Patient Households

\[
\lambda_t^p (1 + \tau_c) - \frac{(1 - \alpha^p c^t)\epsilon^z_t}{c^t - \alpha^p c^t_{t-1}} = 0 \tag{1}
\]

\[
\frac{d^h}{h_t^p} - \lambda_t^p (1 + \tau_h)q^h_t + \beta_t c\left\{ \lambda_t^{p+1} (1 + \tau_h)q^h_{t+1} \right\} = 0 \tag{2}
\]

\[
\lambda_t^p (1 + \tau_{fd}) - \beta_t c\left\{ \lambda_t^{p+1} \left[ \frac{1 + (1 - \tau_d)^r_t}{\pi_t} \right] + \tau_{fd} \right\} = 0 \tag{3}
\]

\[
(1 - \epsilon^h_t)E_t^p \left\{ \lambda_t^{p+1} \left[ \frac{1 + (1 - \tau_d)^r_t}{\pi_t} \right] + \frac{d^p q^h_t + \lambda_t^{p+1} f^p}{\lambda_t^p (1 - \tau_w) w_t} \right\} = 0 \tag{4}
\]

\[
(1 - \tau_w)w_t^p \ell_t^p + \left[ \frac{1 + (1 - \tau_d)^r_t}{\pi_t} + \tau_{fd} \right] d_{t-1}^p + \frac{r_t^h}{\pi_t} - \frac{T_t^w}{\pi_t} \left( \frac{T_t^w}{\pi_t} + \gamma_t \right) + (1 - \omega^b) \frac{E_{t-1}^p}{\pi_t} = 0 \tag{5}
\]

\[
T_t^w = \gamma_p \left( \frac{d_t^w}{2} \right) \left( \frac{T_t^w}{\pi_t} - \frac{T_t^w}{\pi_{t-1}} \pi_t \right)^2 \tag{6}
\]

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1.2 Impatient Households

\[ \lambda_t^i (1 + \tau_c) - \frac{(1 - a^{ci}) c_t^i}{c_t^i - a^{ci} c_{t-1}^i} = 0 \]  
(8)

\[ \frac{a^{bi} c_t^h}{h_t^i} - \lambda_t^i (1 + \tau_h) q_{t+1}^h + \xi_t^i m_t^i E_t \left\{ q_{t+1}^h \pi_{t+1}^i \right\} + \beta_i E_t \left\{ \lambda_{t+1}^i (1 + \tau_h) q_{t+1}^h \right\} = 0 \]  
(9)

\[ \lambda_t^i (1 + \tau_{fb}) - \beta_i E_t \left\{ \lambda_{t+1}^i \left( \frac{1 + r_{bi}^i}{\pi_t+1} - \tau_{fb}^i \right) \right\} - \frac{c_t^i (1 + r_{bi}^i)}{\lambda_t^i (1 + \tau_f^i)} = 0 \]  
(10)

\[ \left[ 1 - \varepsilon_t^i \right] \xi_t^i - \eta \left( \pi_t^{w_i} - \pi_{t-1}^{w_i} \pi_{t-1}^{1-w} \pi_t^{w_i} \right) + a^{bi} \xi_t^{1+w} \xi_t^i + \frac{a^{bi} \xi_t^{1+w} \xi_t^i}{\lambda_t^i (1 + \tau_w)} \pi_t^{w_i} = 0 \]  
(11)

\[ \beta_i E_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[ \eta \left( \pi_{t+1}^{w_i} - \pi_t^{w_i} \pi_{t-1}^{1-w} \pi_t^{w_i} \right) \pi_{t+1}^{w_i} \pi_t^{w_i} \pi_{t-1}^{w_i} \right] \right\} = 0 \]  
(12)

\[ (1 + r_{bi}^i) b_t^i = m_t^i E_t \left\{ q_{t+1}^h \pi_t^{w_i} \pi_t^{w_i} \pi_{t+1}^i \right\} \]  
(13)

\[ (1 + \tau_c) c_t^i + (1 + \tau_h) q_t^h (h_t^i - h_{t-1}^i) + \left( \frac{1 + \tau_{bi}^i}{\pi_t} - \tau_{fb}^i \right) b_{t-1}^i = \]  
(14)

\[ (1 - \tau_w) w_t^i \xi_t^i + (1 - \tau_{fb}^i) b_t^i - \frac{\pi_t^{w_i}}{\tau_t (1 + \tau_f^i + \tau_w^i)} \]  
(15)

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1.3 Hand-to-Mouth Households

\[
\left( \frac{1 - \tau_w}{1 + \tau_c} \right) \left( (1 - \varepsilon_t^m) \ell_t^m + \pi^w_t - \pi^w_{t-1} \pi^{1-iw}_{t-1} \right) \pi^{wm}_t + \frac{a_{t+1} \ell_{t+1} + \phi}{u^m_{t+1}} + \beta_m E_t \left\{ \frac{U^{m+1}_{t+1}}{U^m_{t+1}} \left[ \eta_w \left( \frac{1 - \tau_w}{1 + \tau_c} \right) \left( \pi^{wm}_{t+1} - \pi^w_{t+1} \pi^{1-iw}_{t+1} \right) \pi^{wm}_{t+1} \right] \right\} = 0
\]

(16)

\[(1 + \tau_c) c^m_t = (1 - \tau_w) w^m_t \ell_t^m - \frac{T^u_{m+1}}{\gamma_m}
\]

(17)

\[U^m_{c,t} = \frac{(1 - a_{cm}) \varepsilon_z^m}{\varepsilon_z^m - a_{cm} \varepsilon_z^m_{t-1}}
\]

(18)

\[\pi^w_{t+1} = \left( \frac{w^m_t}{w^m_{t-1}} \right) \pi_t
\]

(19)

\[T_{m+1} = \gamma_m \left( \frac{\eta_w}{2} \right) \left( \pi^{wm}_{t+1} - \pi^{wm}_{t-1} \pi^{1-iw}_{t-1} \right)^2 w^m_t
\]

(20)

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1.4 Entrepreneurs

\[
\lambda^e_t (1 + \tau_c) - \frac{1 - a^{ce}}{c^e_t - a^{ce}c^e_{t-1}} = 0
\]  
(21)

\[
q^k_t = (1 - \tau_k) r^k_t + \beta_e E_t \left\{ \frac{\lambda^e_{t+1}}{\lambda^e_t} \left[ q^k_{t+1} (1 - \delta) \right] \right\} + \left( \frac{\xi^e_t}{\lambda^e_t} \right) m^e_t E_t \left\{ q^k_{t+1} (1 - \delta) \pi_{t+1} \right\}
\]  
(22)

\[
\lambda^e_t (1 - \tau_{fb}) - \xi^e_t (1 + r^{be}_t) - \beta_e E_t \left\{ \lambda^e_{t+1} \left( \frac{1 + r^{be}_t}{\pi_{t+1}} - \tau_{fb} \right) \right\} = 0
\]  
(23)

\[
(1 + r^{be}_t) b^e_t = m^e_t E_t \left\{ q^k_{t+1} \pi_{t+1} (1 - \delta) k^e_t \right\}
\]  
(24)

\[
(1 + \tau_c) c^e_t + \left( \frac{1 + r^{be}_t}{\pi_i} - \tau_{fb} \right) b^e_{t-1} + q^k_t k^e_t =
\]  
(25)

\[
(1 - \tau_k) r^k_t k^e_t + (1 - \tau_{fb}) b^e_t + q^k_t (1 - \delta) k^e_{t-1} + \frac{\beta^e_t}{\tau_e} + \frac{\beta^{be}_t}{\tau_e} - \frac{r^{be}_t}{(\tau_p + \tau_i + \gamma_c)}
\]

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1.5 Intermediate goods producers

\[ w_t^p = \frac{\mu^p(1-\alpha)}{x_t} y_t^{\frac{x}{y}} \]  
\[ w_t^i = \frac{\mu^i(1-\alpha)}{x_t} y_t^{\frac{x}{y}} \]  
\[ w_t^m = \frac{\mu^m(1-\alpha)}{x_t} y_t^{\frac{x}{y}} \]

\[ \psi'(u_t) = A_t \left( \frac{K^{k}_{t-1}}{\gamma_e} \right)^{\alpha_g} \left[ \left( \ell_t^{pp} \right)^{\mu_p} \left( \ell_t^{ii} \right)^{\mu_i} \left( \ell_t^{mm} \right)^{\mu_m} \right]^{1-\alpha} \]  
\[ r_t^k = \beta_e E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \psi'(u_{t+1})u_{t+1} - \psi(u_{t+1}) \right] \right\} \]

\[ y_t^x = A_t \left( k^{ee}_{t-1}u_t \right)^{\alpha} \left[ \left( \ell_t^{pp} \right)^{\mu_p} \left( \ell_t^{ii} \right)^{\mu_i} \left( \ell_t^{mm} \right)^{\mu_m} \right]^{1-\alpha} \left( \frac{K^{k}_{t-1}}{\gamma_e} \right)^{\alpha_g} \]

\[ \frac{f_t^x}{\gamma_x} = \frac{y_t^x}{x_t} - w_t^p \ell_t^{pp} - w_t^i \ell_t^{ii} - w_t^m \ell_t^{mm} - r_t^k k^{ee} - \psi(u_t)k^{ee}_{t-1} - \Phi_x \]

\[ \psi(u_t) \equiv \psi_{u_t}(u_t - 1) + \frac{\psi_{u_t}}{2}(u_t - 1)^2 \]

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1.6 Capital producers and good retailers

\[
q_t^k \left[ 1 - \eta_i \left( \frac{i_t^k \varepsilon_t^k}{i_{t-1}^k} - 1 \right)^2 - \eta_i \left( \frac{i_t^k - 1}{i_{t-1}^k} \right) i_t \right] + \\
\beta_p E_t \left\{ \frac{x_{t+1}}{x_t} \left[ q_t^k \eta_i \left( \frac{i_{t+1}^k \varepsilon_t^k}{i_t^k} - 1 \right) \left( \frac{i_{t+1}^k}{i_t^k} \right)^2 \right] \right\} = 1
\]

\[
k_t - (1 - \delta)k_{t-1} = \left[ 1 - \eta_i \left( \frac{i_t^k \varepsilon_t^k}{i_{t-1}^k} - 1 \right)^2 \right] i_t
\]

\[
\frac{j_t^k}{\gamma_k} = q_t^k \left\{ \left[ 1 - \eta_i \left( \frac{i_t^k \varepsilon_t^k}{i_{t-1}^k} - 1 \right)^2 \right] i_t \right\} - i_t - \Phi_k
\]

\[
1 - e_t^y + \frac{e_t^y}{x_t} - \eta_p \pi_t \left( \pi_t - \pi_{t-1} \pi_{ss}^{-1} \right) + \\
\beta_p E_t \left\{ \frac{x_{t+1}}{x_t} \left[ \pi_t^2 \left( \frac{y_{t+1}}{y_t} \right) \eta_p \left( \pi_t - \pi_{t-1} \pi_{ss}^{-1} \right) \right] \right\} = 0
\]

\[
\frac{j_t^R}{\gamma} = y_t \left[ 1 - \frac{1}{x_t} - \eta_p \left( \frac{y_t \left( \pi_t - \pi_{t-1} \pi_{ss}^{-1} \right)^2 \right) \right]
\]

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1.7 Banks and monetary policy

\[
(1 + r_t) = (1 + r_{ss})^{(1-\phi_r)}(1 + r_{t-1})^{\phi_r} \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi(1-\phi_r)} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y(1-\phi_r)} (1 + e_t)
\]  

(38)

\[
(r_t^b - r_t) = -\eta_b \left( \frac{k^b_t}{b_t^p} - \nu_b \right) \left( \frac{k^b_t}{b_t^p} \right)^2
\]  

(39)

\[
\pi_t k^b_t = \left( \frac{1 - \delta_b}{\epsilon_t^b} \right) k^b_t + \omega_b \left( \frac{\pi_t f_{t-1}^b}{\gamma_b} \right)
\]  

(40)

\[
b_t^b = d_t^b + k^b_t
\]  

(41)

\[
1 + r_t \left( \frac{\theta^d_t}{\theta^d_{t-1}} \right) = \left( \frac{\theta^d_t}{\theta^d_{t-1}} \right) + \eta_p \left( \frac{r^d_t}{r^d_{t-1}} - 1 \right) \frac{r^d_t}{r^d_{t-1}}
\]  

(42)

\[
-\beta_p E_t \left\{ \frac{\lambda^{p+1}_t}{\lambda^p_t} \left[ \eta_p \left( r^d_{t+1} \right)^2 \frac{d^pp_{t+1}}{d^d_{t+1}} \right] \right\} = 0
\]  

(43)

de_0 = 0

d^p_{t-1} = 0

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\[ 1 + \frac{r_i^b}{r_i^t} \left( \frac{\theta_{bi}^t}{r_i^t - 1} \right) - \left( \frac{\theta_{bi}^t}{r_i^t - 1} \right) - \eta_{bi} \left( \frac{r_i^j}{r_i^{j-1}} - 1 \right) \frac{r_i^j}{r_i^{j-1}} + \beta_p E_t \left\{ \frac{\lambda_t^{pi} - r_i^{pi}}{\lambda_t^{pi} - 1} \right\} = 0 \] (44)

\[ 1 + \frac{r_i^b}{r_i^t} \left( \frac{\theta_{be}^t}{r_i^t - 1} \right) - \left( \frac{\theta_{be}^t}{r_i^t - 1} \right) - \eta_{be} \left( \frac{r_i^j}{r_i^{j-1}} - 1 \right) \frac{r_i^j}{r_i^{j-1}} + \beta_p E_t \left\{ \frac{\lambda_t^{pe} - r_i^{pe}}{\lambda_t^{pe} - 1} \right\} = 0 \] (45)

\[ b_i^b - b_i^{ii} - b_i^{ee} - \frac{\theta_{gb}^t}{\theta_{gb}^t - 1} = 0 \] (46)

\[ j_i^t = r_i^{bi} b_i^t + r_i^{be} b_i^{ee} + \theta_{gb}^t \left( \frac{B_i^t}{\theta_{gb}^t} \right) - r_i^{d} d_i - \eta_{d} \left( \frac{B_i^t}{\theta_{gb}^t} - v_{b} \right) \] (47)

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1.8 Fiscal policy

\[ K^g_t = (1 - \delta^g)K^g_{t-1} + I^g_t \]  

\[ \psi^{bg}_t \equiv \left( \frac{B^g_t}{\gamma y_t} \right) \]  

\[ \psi^{cg}_t \equiv \left( \frac{C^g_t}{\gamma y_{ss}} \right) \]  

\[ \psi^{ig}_t \equiv \left( \frac{I^g_t}{\gamma y_{ss}} \right) \]  

\[ \psi^{tg}_t \equiv \left( \frac{T^g_t}{\gamma y_{ss}} \right) \]  

\[ \psi^{bg}_t = (1 - \rho_{cg}^g)\psi^{bg}_{ss} + \rho_{cg}^g\psi^{bg}_{t-1} + \sigma_{cg}^g e^g_t \]  

\[ \psi^{ig}_t = (1 - \rho_{ig}^g)\psi^{ig}_{ss} + \rho_{ig}^g\psi^{ig}_{t-1} + \sigma_{ig}^g e^g_t \]  

\[ \psi^{tg}_t = \psi^{tg}_{t-1} + \rho_{1g1}(\psi^{bg}_t - \psi^{bg}_{ss}) + \rho_{1g2}(\psi^{bg}_t - \psi^{bg}_{t-1}) \]  

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<td>-</td>
<td>78</td>
<td>( \rho_{1g1}, \rho_{1g2} )</td>
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\[
\begin{align*}
C_i^g + I_i^g + \left( \frac{1 + \theta_{ss}^g \pi_{f-1}}{\pi_f} \right) B_{i-1}^g &= B_i^g + T_i^g \\
+ \tau_c \left[ \gamma_p c_i^p + \gamma_i c_i^1 + \gamma_c c_i^l + \gamma_m c_i^m \right] \\
+ \tau_{k_k} [ \gamma_p (h_i^p - h_{i-1}^p) + \gamma_i (h_i^l - h_{i-1}^l) ] \\
+ \tau_w [ w_i^p \gamma_p \ell_i^p + w_i^l \gamma_i \ell_i^l + w_i^m \gamma_m \ell_i^m ] + \tau_{k_k} K_i \\
+ \tau_{fb} \left[ \gamma_p (d_i^p - d_{i-1}^p) + \gamma_i (b_i^l - b_{i-1}^l) + \gamma_c (b_i^c - b_{i-1}^c) \right] \\
+ \tau_d \left( \frac{\ell_{i-1}^d}{\pi_f} \right) \gamma_p d_{i-1}^p 
\end{align*}
\]

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2. Market clearing

\begin{align*}
\gamma_c k_t^c &= \gamma_k k_t & (57) \\
\gamma_x k_t^{ce} &= \gamma_e k_t^c \\
\gamma_x y_t^e &= \gamma_y y_t & (59) \\
\gamma_x \ell_{pp}^i &= \gamma_{p} \ell_{pp}^i \\
\gamma_x \ell_{ii}^i &= \gamma_{i} \ell_{ii}^i & (61) \\
\gamma_x \ell_{mm}^m &= \gamma_{m} \ell_{mm}^m & (62) \\
H &= \gamma_i h_i^i + \gamma_p h_i^p & (63) \\
\gamma_i b_i^i &= \gamma_{b} b_i^i \\
\gamma_i b_i^{ei} &= \gamma_{b} b_i^{ei} & (65) \\
\gamma_p d_t^p &= \gamma_{b} d_t^{pp} & (66)
\end{align*}

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3. Aggregation and equilibrium

\[
K_t \equiv \gamma_k k_t
\]  
(67)

\[
I_t = \gamma_i i_t
\]  
(68)

\[
Y_t \equiv \gamma y_t
\]  
(69)

\[
C_t = \left[ \gamma_p c^p_t + \gamma_i c^i_t + \gamma_c c^c_t + \gamma_m c^m_t \right]
\]  
(70)

\[
B^i_t \equiv \gamma_i b^i_t
\]  
(71)

\[
B^c_t \equiv \gamma_c b^c_t
\]  
(72)

\[
B_t \equiv B^i_t + B^i_t + B^c_t
\]  
(73)

\[
K^b_i \equiv \gamma_p k^b_i
\]  
(74)

\[
D_t \equiv \gamma_d d^b_t
\]  
(75)

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4. Auxiliary equations

\[
\pi_t^\omega \equiv \left[ \frac{\gamma_p w_t^p + \gamma_i w_t^i + \gamma_m w_t^m}{\gamma_p w_{t-1}^p + \gamma_i w_{t-1}^i + \gamma_m w_{t-1}^m} \right] \times \pi_t \quad (76)
\]

\[
Y_t^1 = C_t + I_t + C_t^g + I_t^g \quad (77)
\]

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5. Shocks

\[
\log e_t = (1 - \rho_{ez}) \log e_{ts} + \rho_{ez} \log e_{t-1} + \sigma_z e_t \sim \mathcal{N}(0,1) \tag{78}
\]

\[
\log e_t = (1 - \rho_{eh}) \log e_{th} + \rho_{eh} \log e_{t-1} + \sigma_h e_t \sim \mathcal{N}(0,1) \tag{79}
\]

\[
\log e_t = (1 - \rho_{el}) \log e_{tel} + \rho_{el} \log e_{t-1} + \sigma_{el} e_t \sim \mathcal{N}(0,1) \tag{80}
\]

\[
\log m_i = (1 - \rho_{mi}) \log m_{is} + \rho_{mi} \log m_{i-1} + \sigma_{mi} e_{mi} \sim \mathcal{N}(0,1) \tag{81}
\]

\[
\log m_i = (1 - \rho_{me}) \log m_{es} + \rho_{me} \log m_{e-1} + \sigma_{me} e_{me} \sim \mathcal{N}(0,1) \tag{82}
\]

\[
\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \sigma_A e_t^A \sim \mathcal{N}(0,1) \tag{83}
\]

\[
\log e_t = (1 - \rho_{ek}) \log e_{ks} + \rho_{ek} \log e_{k-1} + \sigma_k e_t \sim \mathcal{N}(0,1) \tag{84}
\]

\[
\log e_t = (1 - \rho_{ey}) \log e_{ys} + \rho_{ey} \log e_{y-1} + \sigma_y e_t \sim \mathcal{N}(0,1) \tag{85}
\]

\[
\log e_t = (1 - \rho_{ek}) \log e_{ks} + \rho_{ek} \log e_{k-1} + \sigma_k e_t \sim \mathcal{N}(0,1) \tag{86}
\]

\[
\log \theta_t = (1 - \rho_{\theta d}) \log \theta_{sd} + \rho_{\theta d} \log \theta_{d-1} + \sigma_d e_t \sim \mathcal{N}(0,1) \tag{87}
\]

\[
\log \theta_t = (1 - \rho_{\theta h}) \log \theta_{sh} + \rho_{\theta h} \log \theta_{h-1} + \sigma_h e_t \sim \mathcal{N}(0,1) \tag{88}
\]

\[
\log \theta_t = (1 - \rho_{\theta e}) \log \theta_{se} + \rho_{\theta e} \log \theta_{e-1} + \sigma_e e_t \sim \mathcal{N}(0,1) \tag{89}
\]
6. Total resources constraint

\[
Y_t = C_t + I_t + C_t^S + I_t^S + \left(\psi_{u_t}(u_t - 1) - \frac{\psi_{u_t}}{2}(u_t - 1)^2\right) K_{t-1} + \frac{\delta_b K_{t-1}^b}{\pi_t} + \frac{\eta_p}{2} \left(\pi_t - \pi_{t-1}^{\eta_p} \pi^{1-\eta_p} \right)^2 Y_t
\]

\[
+ \frac{1}{\pi_t} \left[ \frac{\eta_d}{2} \left( r_{t-1}^d - r_{t-2}^d \right)^2 D_{t-1} + \frac{\eta_{r_i}}{2} \left( r_{t-1}^{r_i} - r_{t-2}^{r_i} \right)^2 B_{t-1}^i + \frac{\eta_p}{2} \left( r_{t-1}^p - r_{t-2}^p \right)^2 B_{t-1}^p \right]
\]

\[
+ \frac{\eta_d}{2} \left( K_{t-1}^b - v_b \right)^2 + \frac{\eta_{r_i}}{2} \left( K_{t-1}^{r_i} \right) + \frac{\gamma_w}{2} \left( \pi_t^{w_p} - \pi_{t-1}^{w_p} \pi^{1-w_p} \right)^2 w_t^p + \frac{\gamma_{w_i}}{2} \left( \pi_t^{w_i} - \pi_{t-1}^{w_i} \pi^{1-w_i} \right)^2 w_t^i
\]

\[
+ \frac{\gamma_w}{2} \left( \pi_t^{w_m} - \pi_{t-1}^{w_m} \pi^{1-w_m} \right)^2 w_t^m
\]
References


